

# Collaboration in Mathematics Teacher Education: the What, How, and Why of Mathematical Modeling

**Aubrey Neihaus, Wichita State University**

*Aubrey Neihaus is an assistant professor in the School of Education at Wichita State University. She is the Program Chair for the Middle and Secondary Mathematics Teacher Education programs. Her research is in mathematics teacher education, with foci on collaboration, and teacher learning around equity and social justice.*

**Amy Been Bennett, University of Nebraska-Lincoln**

*Amy Bennett is a postdoctoral research associate in the Department of Mathematics at the University of Nebraska-Lincoln. Her research involves mathematical modeling, elementary and secondary teacher education, and equitable teaching practices.*

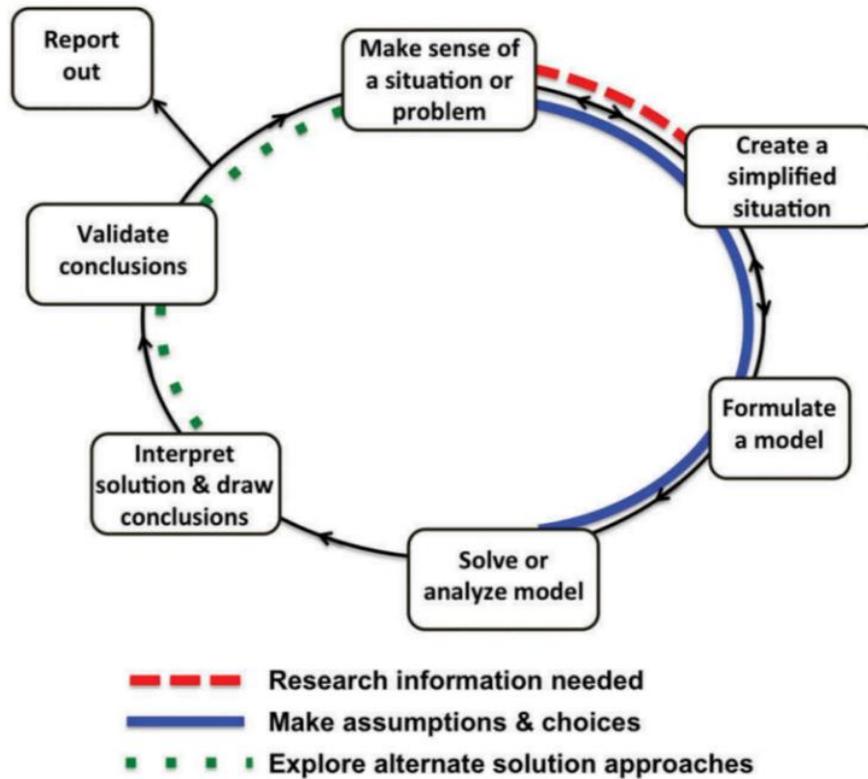
## Introduction

Mathematical modeling is important in mathematics teacher education. In this reflection paper, we share our collaboration in developing and implementing a mathematical modeling task for prospective secondary mathematics teachers. This collaboration was between two universities, across mathematics and mathematics education, and the task was implemented in a mathematics content course and a mathematics pedagogy course. The purpose of this paper is to outline the *what*, *how*, and *why* of mathematical modeling in mathematics teacher education, as well as what makes it a particularly fruitful site for collaboration between mathematics and mathematics education.

## Mathematical Modeling

Mathematical modeling is a process by which mathematics is used to make sense of phenomena in the world (KSDE, 2019; NSBE, 2015). The process is iterative, as the model is analyzed for its alignment to the phenomenon, and then adjusted to better explain or predict the phenomenon. In this way, the modeling process offers opportunities for revisions in the learner's thinking process and model. Figure 1 (Anhalt, Cortez, & Bennett, 2018) offers one illustration of the modeling process as a cycle. This diagram highlights how the learner navigates between the "mathematical world" (creating and solving a model) and the "real world" (interpreting and validating the model).

**Figure 1:** The Modeling Cycle



In Figure 1, the cycle begins at the top of the diagram, with “Make sense of the situation and simplify it.” The learner will then “Formulate a model” that will be used to explain or predict the problem. This is the first version of the model. Note that the relationship between the first stage and the second is bi-directional, meaning that the learner might move back and forth between these two stages before moving to the third. When the learner feels that the model fits the situation and is ready to be tested, they then move on to “Solve or analyze the model.” This stage might include calculations in solving an equation, mapping out geometric areas to consider overlap, or another mathematical process, depending on the mathematics inherent in the model. The information gained in this stage is then used in the following stage, “Interpret solution and draw conclusions.” At this point in the modeling cycle, the learner determines what the outcome of the model means in terms of the real world situation. They then use this information to determine whether the model is sufficiently valid, in the “Validate conclusions” stage of the cycle. At this stage, there are two routes the learner might take: if the model is sufficiently valid, they will report out their conclusions and their model; if the model is not sufficiently valid, they will return to the situation they are modeling, to consider where their model was not sufficiently sensitive to the details of the situation. If the latter, they repeat the modeling cycle to develop another iteration of the model.

## Modeling and K-12 Education

While mathematical modeling has been a long-standing domain of mathematics research and undergraduate study, it was not explicitly included in the K-12 standards until the most recent standards reforms (Anhalt & Cortez, 2016; KSDE, 2019; NSBE, 2015). Within the standards,

modeling is unique as it is both a practice standard and a content domain. However, as a content domain it does not stand alone, but intersects with each of the other domains in the high school standards. In this way, modeling is both *how* to engage in mathematical thinking, and *what* mathematics content to consider. Stated another way, the *how* of modeling is evident through use of the modeling cycle; the *what* of modeling is evident in the use of other domains of mathematics to build the model.

To meet recent standards reforms that explicitly incorporate modeling, mathematics teacher educators and researchers developed tasks to support the learning of modeling for both teachers and students. To support the modeling cycle, tasks must be open-ended and provide authentic contexts so that students have an opportunity to make sense of the phenomenon under study and solve the real-world problem.

Some work has foregrounded what content is amenable to modeling, such as a school bake sale to support modeling of polynomials in the real world (Baron, 2015). Work like this helps us to expand modeling beyond linear equations. Other work (e.g., Anhalt & Cortez, 2015) foregrounds both the pedagogy and the mathematical sense-making process of modeling in the classroom. Combined, this work offers teachers support in *what* content might be used in modeling and *how* to implement modeling in the classroom.

There is also the *why* of modeling: modeling has the potential to open up contexts that valorize students' funds of knowledge (Moll et al., 1992) or take up social issues (Anhalt, Staats, et al., 2018). Felton and colleagues (2015) asked students to develop a model to determine whether a shower or a bath is more effective for water conservation. Similarly, the Flint Water Task (Aguirre et al., 2019) used the issue of tainted municipal water in Flint, Michigan and solutions proposed by corporations (donating bottled water). In this task, students used modeling to determine how many bottles of water a student would need daily to replace municipal water, and whether the corporations' solution was a good one. These tasks created space for rich discussion about social problems and how mathematics can be used to both propose and analyze solutions.

## The Sprinkler Task

Building on the work referenced above, we sought to design a modeling task that considered the *what*, *how*, and *why* of mathematical modeling (see Bennett & Neihaus, 2022). The Sprinkler Task is shown in Figures 2 and 3. When we implemented the task in our mathematics methods course (Wichita State University) and mathematics content course (University of Nebraska-Lincoln), we also provided a schematic of the backyard on grid paper and a table of available equipment. The schematic had irregular shapes for the flower gardens and labeled three berry bushes, a sapling, and two mature trees. The table of equipment included photographs of hoses, sprinklers, timers, and splitters, all taken at a local garden center, all with descriptions of features and price visible.

### Figure 2: The Sprinkler Task Prompt

Your elderly neighbor has asked for your help. She enjoys gardening but wants to purchase a low-maintenance (and relatively inexpensive) system for watering her yard and gardens. She plans to get her supplies from the local gardening store and have you set it up for her.

Use the backyard schematic and the list of available gardening equipment that your neighbor provided from her recent trip to the gardening store to create a plan for how to position the watering equipment in her backyard.

Consider how you will indicate where the various gardening equipment should be placed on the schematic and show your neighbor how different parts of the yard will be watered. You can use words, drawings, and mathematical tools in your plan.

### **Figure 3: Opportunity for Revision in the Task**

#### **New information that causes you to revise the model:**

In consultation with your neighbor, you realize that she wants this sprinkler system to be automated because she is going to be away for a month to visit grandchildren. She still needs the same features watered but would like the sprinklers to have a set position and turn on with a timer. How does this new information alter your model?

In designing the Sprinkler Task, we wanted to give students a context where multiple mathematics content domains were relevant (e.g., the two-dimensional geometry of sprinkler coverage; the algebra of materials cost). This required students to make choices about which mathematics to foreground and to consider alternate choices made by other students. The schematic on grid paper and the requirement for precisely communicating the plan were designed so that students might engage with measurement as well (i.e., the coordinate plane). In this way, we endeavored to create a modeling task where *what* content was used in the model was not pre-determined by the task itself, but by the active choices students made.

To support the cyclical nature of the model, we intentionally created the opportunity for revision in Figure 3 and shared it with students after they had created their initial models. This feature of the task created the need to engage in the cyclical aspects of the modeling cycle, ensuring that students were authentically engaging the stages involving analysis, conclusions, and validation. This aspect of the task ensured that *how* students made sense of the problem required the full modeling cycle.

Given our shared context in the Great Plains, and the relevance of water conservation to sustainability and agriculture (e.g., Buday et al., 2021; USDA, 2018), we chose to situate our task in water usage for gardening. Since water conservation is a local issue, this context would be a familiar social issue. Additionally, the agrarian/landscaping context provides a funds-of-knowledge opportunity for students to see and value mathematics in blue-collar professions, rather than merely as the milieu of white-collar professions like finance or engineering. To address the *why* of this modeling task, we chose a context to support prospective teachers in thinking beyond the generalized contexts of nationalized curriculum. We believe there is power in offering tasks that consider local issues and contexts that leverage students' funds of knowledge, particularly for those who may not often see themselves or their families reflected in the curriculum.

### **Reflection**

Through our collaboration in developing and implementing the Sprinkler Task in a mathematics methods course as well as a mathematics content course, we realized that the nature of modeling is particularly fruitful for collaboration between mathematics education and mathematics. This is because quality modeling tasks simultaneously require that teachers understand: 1) *what* math is taught (the content), 2) *how* sense is made of the phenomenon (the full modeling cycle), and 3) *why* this might be compelling or empowering for students (relevant contexts). This is not merely due to its place in the standards, but because of the cross-disciplinary nature of mathematical modeling. In developing and implementing this task in a mathematics content course and a math methods course, we found these aspects to be inherently bound up with one another. We are convinced that one cannot consider a modeling task to be high quality without any one of these three components, and these three components require expertise and knowledge from both the fields of mathematics and mathematics education.

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