

The Baltic International Yearbook of
Cognition, Logic and Communication

November 2013

pages 1-31

Volume 8: *Games, Game Theory
and Game Semantics*
DOI: 10.4148/1944-3676.1079

BENOÎT CASTELNÉRAC
Université de Sherbrooke

MATHIEU MARION
Université du Québec à Montréal

ANTILOGIC

ABSTRACT: This paper is an interim report of joint work begun in Castelnérac & Marion (2009) on dialectic from Parmenides to Aristotle. In the first part we present rules for dialectical games, understood as a specific form of *antilogikê* developed by philosophers, and explain some of the key concepts of these dialectical games in terms of ideas from game semantics. In the games we describe, for a thesis A asserted by the answerer, a questioner must elicit the answerer's assent to further assertions B_1, B_2, \dots, B_n , which form a scoreboard from which the questioner seeks to infer an impossibility (*adunaton*); we explain why the questioner must not insert any of his own assertions in the scoreboard, as well as the crucial role the Law of Non Contradiction, and why the games end with the inference to an impossibility, as opposed to the assertion of $\neg A$. In the second part we introduce some specific characteristics of Eleatic Antilogic as a method of enquiry. When Antilogic is used as a method of inquiry, then one must play not only the game beginning with a given thesis A , but also the game for $\neg A$ as well as for $A \ \& \ \neg A$, while using a peculiar set of opposite predicates to generate the arguments. In our discussion we hark back to Parmenides' *Poem*, and illustrate our points

with Zeno's arguments about divisibility, Gorgias' ontological argument from his treatise *On Not-Being*, and the second part of Plato's *Parmenides*. We also identify numerous links to Aristotle, and conclude with some speculative comments on the origin of logic.

In the twentieth century, Paul Lorenzen was the first to suggest the idea of a game semantics in 'Logik und Agon' (Lorenzen 1960). As the title indicates, Lorenzen had in mind the practice of the *eristikê* or *dialektikê* of the ancient Greeks. Since Socrates, these words came to acquire a variety of meanings, but they used to designate verbal bouts between two players, a questioner and an answerer, in front of an audience, possibly with a referee. Another common name for this practice was *antilogikê* or *antilogic*, which we chose for our title. In Plato and Aristotle (and others before them), *antilogikê* refers to the practice of contradicting, and thus equally applies to long rhetorical speeches with opposite conclusions or the practice of arguing through short questions and answers to lead an opponent into contradicting himself. In the latter case, the answerer would begin with an assertion A —often, in Plato's dialogues, as an answer to a question of the form 'What is X ?' raised by the questioner—and they would proceed, taking turns, with the questioner asking short questions with yes/no answers from the answerer. The aim of the questioner was to elicit from the answerer assent to a number of further assertions B_1, \dots, B_n from which he would infer $\neg A$, thus showing that the answerer holds an inconsistent set of beliefs that include A . Typically, the questioner would win the bout by driving his opponent into a contradiction; the answerer would win by avoiding being thus caught holding a contradictory pair of claims. This procedure, called the *elenchus*, was intended as a *test*, and it is sometimes referred to in the secondary literature as the 'elenctic method' or, more confusingly, the 'Socratic elenchus'.¹

We contend that, already in Eleatic circles, philosophers used the art of contradicting, i.e., 'antilogic', as a method of enquiry. The social and political conditions under which the Greeks thus began regimenting public debates in this manner are well worth an investigation, but this is not our aim in this paper. It is worth noting, however, that our modern practice of cross-examination in law courts did not exist in Ancient Greece, where pleading occurred in two opposing speeches.

(Nevertheless, as we shall see, philosophers were wont to point out the importance of the Principle of Non Contradiction in such contexts.) But the practice was widespread, as one can see from the bemused report of one of the authors of the *Hippocratic Corpus*:

He who is accustomed to hear speakers discuss the nature of man beyond its relations to medicine will not find the present account of any interest. For I do not say at all that a man is air, or fire, or water, or earth, or anything else that is not an obvious constituent of man; such accounts I leave to those that care to give them. Those, however, who give them have not in my opinion correct knowledge. For while adopting the same idea they do not give the same account. Though they add the same appendix to their ideas—saying that ‘what is’ is a unity, and that this is both unity and the all—yet they are not agreed as to its name. One of them asserts that this one and the all is air, another calls it fire, another, water, and another, earth; while each appends to his own account evidence and proofs that amounts to nothing. The fact that, while adopting the same idea, they do not give the same account, shows that their knowledge too is at fault. *The best way to understand this is to be present when they contradict (auteoisin antilegousin). Indeed, when the same speakers contradict one another (pros allêlous antilegontes hoi autoi) in front of the same audience, the same man never prevails in the debate (perigignetai en tōi logōi) three times in succession, but now one is victor, now another, now he who happens to have the most glib tongue in the face of the crowd.* Yet it is right that a man who claims correct knowledge about the facts should maintain his own argument victorious always, if his knowledge be knowledge of reality and if he set it forth correctly. But in my opinion such men by their lack of understanding overthrow themselves in the words of their very discussions, and establish the theory of Melissus.² (Our italics.)

Without naming them, the author clearly describes the use of contradictory arguments, using the key word *antilegontes*, ‘they contradict’ in the italicized part. In what follows, we wish merely to propose the

outline of a reconstruction of the practice of ‘*antilogikê*’, in the time span from Parmenides to Aristotle, i.e., from the 5th to the 4th century B.C.E.. The reference here to Eleatic ideas, and Melissus in particular, is particularly relevant for our purpose.

It goes without saying that every single historical claim we are about to make is contested, but it is not our aim to argue in detail for any of them here, over and above giving some brief textual indications and some references to the secondary literature. Furthermore, we do not wish to present a ‘model’ for *antilogikê* in any logical or mathematical sense of the word (and obviously not in the sense of model-theoretical semantics). Our aim is to provide a reconstruction of a practice, which can be said to provide a picture and, in that ordinary sense only, a ‘model’. Although game semantics is guiding this reconstruction, we wish to steer clear of any specific approach.³ We still hope that the following can be used as a template for further modifications—this being merely an interim report for our own investigations—as well as for further logical and linguistic investigations.⁴ In point of fact, we think that the following gives an illustration of the value of game semantics in the study of the history of logic.

In the first part of the paper, we will give a set of rules for dialectical games,⁵ which will serve as our point of departure. In the second part, we will look at some specific characteristics of what could be the origin of these games, i.e. a peculiar form of inquiry by means of contradicting arguments that clearly has its source in Parmenides’ *Poem*. We will conclude with some speculation about the origin of logic.

1. DIALECTICAL GAMES

Our task of reconstructing the dialectical bouts we briefly described above is extremely difficult, and our conclusions rather conjectural, for a number of reasons, one of which being that so much that was written has been lost.⁶ For example, Diogenes Laertius gives a series of titles by Protagoras that include one treatise on *The Art of Controversy* (*Tekhnê eristikôn*), another called *Of Forensic Speech for a Fee*, and two *Books of Opposing Arguments*.⁷ At the very end of *On Sophistical Refutations* (34, 183b34-36), Aristotle also mentions the Sophists:

[...] the training given by the paid professors of contentious arguments was like the practice of Gorgias. For he used to hand out rhetorical speeches to be learned by heart, and they handed out speeches in the form of question and answer, which each supposed would cover most of the arguments on either side.⁸

These are also lost. We are fortunate to have *Topics* and *On Sophistical Refutations*, taken together, as a complete handbook.⁹ But one has to regret the loss of any prior handbook that would have given us a better idea of the practice prior to Aristotle. This loss also made acute by the fact that Aristotle may have introduced some subtle distortions in his own handbook.

Perhaps the fragment known as *Dissoi Logoi*, which survived attached to Sextus Empiricus, if it is to be dated to the 380s or before, would provide an example of such handbooks, since it partly fits the description from Aristotle quoted above.¹⁰ Its first part is composed of a series of four ‘double arguments’ where opposite conclusions are argued for concerning the identity or distinctness of pairs of qualities, e.g., ‘good’ and ‘bad’, ‘true and false’, followed by a series of single speeches on topics such as ‘Is Virtue teachable?’—a topic that would exercise Plato’s Socrates later on.¹¹ But this is not yet what one is looking for. It is clear that these ‘double arguments’ stand opposed to the epideictic orations (from the Greek *epideixeis*, ‘demonstrations’) of the rhetoricians, but they do not proceed by short questions with yes/no answers. The contrast is made plain in Plato’s *Gorgias*, when Socrates forces the Sophist to answer his questions briefly:

SOCRATES: Well now Gorgias, would you be willing to complete the discussion in the way we’re having it right now, that of alternately asking questions and answering them, and to put aside for another time this long style of speechmaking like the one Polus began with?

GORGAS: There are some answers, Socrates, that must be given by way of long speeches. Even so, I’ll try to be as brief as possible. This, too, in fact is one of my claims. There’s no one who can say the same things more briefly than I.

SOCRATES: That’s what we need Gorgias! Do give me a

presentation of this very thing, the short style of speech, and leave the long style for some other time. (*Gorgias*, 449b-c)¹²

The contrast here is between ‘*makrologia*’ (long style) and ‘*brakhulogia*’ (short style),¹³ and Socrates explicitly links the latter to the ‘method of the *elenchus*’. In *Topics* VIII, 2, 158a15-17, Aristotle also stated that answers should be short and, ideally, ‘yes’ or ‘no’.¹⁴ This fully corresponds to the meaning of ‘*antilogikê*’ in Plato, e.g., not only here, but in a good number of passages on dialectic (in ‘early’ dialogues like *Protagoras* 335a, *Gorgias* 481d, and *Lysis* 216a, but also dialogues in Plato’s ‘middle-period’ like *Phaedrus* 261d, *Republic* 454a & 539b). There are of course many valuable illustrations of this practice in most of Plato’s dialogues, including the early ‘Socratic’ ones, and even in middle and late dialogues, e.g., in Book I of *Republic*, but the dialogues are often interrupted, and the answerer does not always keep to short answers; there are also long speeches by both Socrates and by his opponent, including references to myth, and so forth. It is nevertheless in Plato that we find the best example of dialectical exchanges, i.e., the longest stretch of uninterrupted series of short questions and answers, in the second half of *Parmenides*: in nearly thirty Stephanus pages, from 137c to 166c, a young Aristotle—a character not supposed to represent the philosopher—provides 532 short answers to questions by *Parmenides*.¹⁵ The chains of questions and answers form about 180 arguments, commonly classified in eight or nine series of ‘deductions’. Given the extreme complexity of this argument, it is rather unsurprising that no consensus has emerged amongst contemporary commentators about the very point of this second half.¹⁶

Plato’s dialogues do not merely contain numerous illustrations of dialectic, they also include many instances of participants breaking off the exchange to launch into precious ‘meta-discussions’, so to speak, reflecting on the way they proceed, e.g., in the case of a crucial methodological passage at *Parmenides* 135d-136c (to which we shall return) to give instruction on how to structure a set of dialectical bouts around a given thesis. With the help of these and of Books I and VIII of *Topics*, coupled with *On Sophistical Refutations*, we came up with the following rules:

1. Games always involve two players, which we will name accord-

ing to their initial roles: *Questioner* and *Answerer*, although this is grammatically infelicitous.

2. A play begins with *Questioner* eliciting from *Answerer* his commitment to an assertion or thesis A .
3. The play then proceeds through a series of alternate questions and answers. *Questioner* asks questions such that *Answerer* should give short answers, ideally ‘yes’ or ‘no’.
4. In this way, *Questioner* elicits commitments from *Answerer* to further assertions B_1, B_2, \dots, B_n , which form, taken together with A , *Answerer*’s scoreboard:

$$\{A, B_1, B_2, \dots, B_n\}.$$
¹⁷
5. *Questioner* infers from B_1, B_2, \dots, B_n an impossibility (*adunaton*), namely that $\neg A$.
6. If *Questioner* has driven *Answerer* into a contradiction, the play ends with *Questioner* winning; *Answerer* wins by avoiding being driven into a contradiction. (Winning rule)

The whole procedure is the *elenchus*, properly speaking. Plato uses the term ‘*sullogizesthai*’ at , meaning ‘taking together’ or ‘adding up’, on at least one occasion:

Join me, then, in adding up (*sullogisai*) what follows for us from our agreements. (*Gorgias* 498e)

Likewise for Aristotle, for whom ‘*sullogismos*’ is meant to refer to the *act* of inferring the conclusion itself, and not to the whole ‘*sylogism*’.¹⁸ A logical ‘compulsion’ is certainly felt, e.g., when, at the very end of *Lesser Hippias* Socrates infers that someone who commits injustice voluntarily is a better person than someone who commits injustice involuntarily:

HIPPIAS: I can’t agree with you in that Socrates.
 SOCRATES: Nor I with myself. But given the argument, we can’t help having it look that way to us, now, at any rate.
 (*Lesser Hippias* 376b-c)

This compulsion can also be illustrated by a notorious passage towards the end of *Gorgias* where Socrates refers to the conclusions that have been reached as “bound by arguments of iron and adamant” (508e).

It is worth briefly digressing here on Aristotle’s definition of ‘*sylogism*’ (translated here as ‘*deduction*’):

A *deduction* is a discourse in which, certain things having been supposed, something different from the things supposed results of necessity, because the things are so. (*Prior Analytics* A 1, 24b18-20)

Aristotle never defines this notion of ‘*resulting*’ or ‘*following of necessity*’, all he ever does is to present moods of the first figure as ‘*perfect*’ or ‘*complete*’ *sylogisms*, because in their case nothing needs to be added “for the necessity to be evident” (A 1, 24^b 22-25), and shows afterwards how to reduce the moods of the other figures to those. Because of this *intuitive* recognition of the validity of some basic cases and because his use of schematic letters is best understood in terms of the pre-semantic idea of interpretation by replacement,¹⁹ it would be wrong to attribute to Aristotle a model-theoretical notion of logical consequence.²⁰ Incidentally, it would also be wrong to think, as does Terry Penner, that the lack of the full apparatus of model-theoretic semantics implies that there is no *elenchus* deserving of the name (Penner 2007, 18). It is merely that this apparatus is not suited to the study of antilogic.

That the word used is *adunaton* or ‘*impossibility*’, as opposed to, say, our modern ‘*absurdity*’, merely reflects the fact that it is not possible to think both A and $\neg A$; Aristotle argues this very point in *Metaphysics* IV, 3 1005^b 11-34.²¹ The Principle of Non Contradiction (PNC) is vital to antilogic, and it should not be surprising to find frequent reference to it. As we shall see in the next section, it has its sources in Parmenides’s *Poem*. So far as we know, the earliest straightforward expression of it is in *Gorgias*’ *Defence of Palamedes*:

You have accused me in the indictment we have heard of two most contradictory things, wisdom and madness, things which cannot exist in the same man. When you claim that I am artful and clever and resourceful, you are accusing me of wisdom, while when you claim that I betrayed Greece, you accused me of madness. For it is mad-

ness to attempt actions which are impossible, disadvantageous and disgraceful, the results of which would be such as to harm one's friends, benefit one's enemies and render one's own life contemptible and precarious. And yet *how can one have confidence in a man who in the course of the same speech to the same audience makes the most contradictory assertions about the same subjects?* (Our italics)²²

Gorgias' point is that he who asserts A and $\neg A$ in front of a court, "in the course of the same speech to the same audience", loses all credibility. The same point is made in Plato's *Apology*, when Socrates replies to his cross examination:

You cannot be believed, Meletus, even, I think, by yourself. The man appears to me, men of Athens, highly insolent and uncontrolled. He seems to have made his deposition out of insolence, violence and youthful zeal. He is like one who composed a riddle and is trying it out: "Will the wise Socrates realize that I am jesting and contradicting myself, or shall I deceive him and others?" I think he contradicts himself in the affidavit, as if he said: "Socrates is guilty of not believing in gods but believing in gods", and surely that is the part of a jester.

Examine with me, gentlemen, how he appears to contradict himself, and you, Meletus, answer us. (*Apology* 26e-27b)

There are further statements of the Principle in *Gorgias* 495e-496a, *Republic* 436e-437a, 602e or *Sophist* 230b, and in Aristotle, in *Prior Analytics* I 46, 52a 2-3 and in *Metaphysics* IV, 3, 1005b 23, where it reads:

[...] the same attribute cannot at the same time belong and not belong to the same subject in the same respect.

Aristotle is notorious for the arguments he provides in support of PNC in *Metaphysics* IV, 3–6, including in *Metaphysics* IV, 4 a transcendental argument of sorts, involving the context of antilogic: he points out that either a person asserts something meaningful, e.g., that some x has

property F , or says nothing, but to say nothing is no basis for reasoning, and to use one's meaningful assertion in reasoning implies having to defend it qua *Answerer*; doing this, however, presupposes that the assertion stands opposite to its contrary (that x does not have the property F). In other words, to assert is to be open to a challenge to defend one's assertion, and PNC is presupposed in any attempt to defend it. So PNC is a condition of the possibility of any meaningful assertion.

There is an ambiguity concerning what the *elenchus* is supposed to achieve. Under our view, in driving *Answerer* into a contradiction, *Questioner* merely proved that the set of beliefs *Answerer* has committed himself to is inconsistent. But the usual view, pioneered by Gregory Vlastos in the case of Socrates, has it that, in deriving from *Answerer's* scoreboard that $\neg A$, Socrates believed that, qua *Questioner*, he had shown that $\neg A$ is true and A false (Vlastos 1994, 11). This is obviously problematic, and led Vlastos to speak of a 'problem of the Socratic elenchus' (Vlastos 1994, 3–4). Vlastos made no claim concerning other putative players, only about Socrates, and, by extension, Plato in the 'early' dialogues. This is not the place for a discussion of Vlastos' textual basis.²³ Our fundamental objection to this view, over and above the fact that it attributes an elementary logical mistake to Socrates, is that it overlooks the fact that theses are to be included in *Answerer's* scoreboard not because they are true or known to be true, but merely because *Answerer* asserted or conceded them, i.e., because they are his *beliefs*. To put it differently, a move from

$$A, B_1, B_2, \dots, B_n \vdash \perp$$

to

$$B_1, B_2, \dots, B_n \vdash \neg A$$

is unobjectionable, but in the context of a dialectical game, even if this is an appropriate description of the *elenchus*, there would be no compelling reason to think that one should discard A rather than any of B_1, B_2, \dots, B_n .²⁴ This is a rather remarkable fact that Plato is aware of, as one can see from *Charmides* 164c-d, where Critias, as *Answerer* objects to Socrates:

But this [...], Socrates, would never happen. And if you think it necessary to draw this conclusion from what I admitted before, then I would rather withdraw some of my

statements, and would not be ashamed to admit I made a mistake, in preference to conceding that a man ignorant of himself could be temperate.

What we have here is a case where *Answerer* prefers to withdraw one of B_1, B_2, \dots, B_n rather than to concede $\neg A$.

These last remarks bring to the fore the key fact that it is *Answerer's* beliefs that are tested for consistency. This makes it all the more important that *Questioner* not surreptitiously introduce an additional thesis in *Answerer's* scoreboard, since then *Answerer* could avoid commitment to $\neg A$ by simply denying the thesis that *Questioner* had inserted. The game would not have been a test of his beliefs, hence the necessity for this key rule:

7. *Questioner* may not introduce any thesis, *Answerer* must commit himself to any thesis used by *Questioner*. (Socratic Rule)

We call it the ‘Socratic Rule’ for the simple reason that it has clear motivation within Plato’s dialogues, since it explains both Socrates’ notorious ‘avowals of ignorance’, not only at the outset but also in the middle of the game, e.g., in *Lesser Hippias* 372b-e, where its function is to enforce the ‘doxastic’ or ‘say what you believe’ constraint on *Answerer* (e.g., at *Protagoras* 331c-d, *Gorgias* 472b-c & 550d, *Charmides* 166d-e, or *Republic* 346a).²⁵ The Socratic Rule bears some resemblances with the Formal Rule in dialogical logic, this is why we were at first interested in the study of dialectic from that point of view. Given that we have not given any details about dialogical logic here, we simply refer the reader to our earlier work.²⁶

Among other rules, brief mention should be made of the following:

8. There is a limit on the length of plays.
9. Delaying tactics are forbidden.
10. Fallacies are forbidden.

A passage at *Theaetetus* 172d-e suggests that philosophers, as opposed to those who plead in law courts “with one eye on the clock”, did not limit the length of plays. Yet Aristotle has two passages where the existence of a practice of playing “against the time” is implied, e.g.,

Topics, VIII, 10, 161^a9-12, where he clearly forbids playing against time with inappropriate objections:

The fourth and poorest kind of objection is the one with respect to time: for some people raise the sort of objection that it would take longer to argue against than the present discussion allows.²⁷

Related to this is a ‘no delaying tactics’ rule at *Topics*, VII, 2, 158^a25–30. As for fallacies, it is obvious that ‘apparent’ inferences to a contradiction must be forbidden, and Aristotle wrote *On Sophistical Refutations* to teach his students how to spot these. It is quite remarkable that his list of fallacies was almost fully illustrated already in the *Euthydemus*, which might have served the same pedagogical purpose within the Academy.

In modern game semantics such as dialogical logic, one finds alongside structural rules a specific set of rules for the logical particles. We do not give any here because none were made explicit until later, maybe for the simple reason that they were naturally followed, as they appear linked with interaction within a dialogue; what is truly needed to further our understanding here is a proper account at the level of linguistic theory of the interactive workings of dialogue.²⁸

It is nevertheless worth pointing out the rule for induction (*apagogê*) at *Topics*, VIII, 2, 157^a34-157^b2:

When it happens that, after you have induced from many cases, someone does not grant the universal, then it is your right to ask him for an objection. However, when you have not stated that it does hold of some cases, you have no right to ask ‘of which cases does it not hold?’ For you must previously carry out an induction to ask for an objection in this way.²⁹

The rule states that *Questioner* can introduce a universal proposition only after having had *Answerer* concede instances, and that, once the universal proposition is introduced, *Answerer* must in turn either grant it or provide a putative counterexample. This is needed given that, even with *Answerer's* assent for few instances, there is no logical compulsion to concede the universal proposition.

This rule is illustrated a number of times in Plato's dialogues, e.g., in *Lesser Hippias* 366c-369b. Aristotle was very much aware of this, as he explicitly mentioned this dialogue in *Metaphysics*, V, 29, 1025a6-13.³⁰ This rule is of fundamental importance for him, since he derived from the explanation of universal affirmative propositions that plays a central role in his syllogistic:

We use the expression 'predicated of every' when none of the subject can be taken of which the other term cannot be said. (*Prior Analytics*, A 2, 24^b28-29)

With help of the square of oppositions, it is possible to extend this to the other three basic forms of propositions implicated in his syllogistic rules.³¹

2. ELEATIC ANTILOGIC

The preceding observations were meant as an analysis of a single bout concerning a single thesis *A*, and might be of help in understanding how some of Plato's dialogues, especially the early 'Socratic' ones, proceed. One should, however, distinguish this method of testing from antilogic, conceived now as a *method of inquiry* into problems for which no empirical method could provide an answer. This method of antilogic involves testing not only *A* but also its contradictory $\neg A$.³² Only one set of four arguments has survived extensively enough to exhibit this structure: Parmenides' B8 on the predicates of Being, Zeno's arguments about divisibility, Gorgias' ontological argument at the beginning of his treatise *On Not-Being*, and the remarkable second part of Plato's *Parmenides*.³³ In this section, our intention is not to address the substantial issues these arguments raise, e.g., concerning the semantics of not-being, even less so to assess them; we would merely like to describe some of the characteristics these arguments exhibit. This section may thus be less interesting for logicians or linguists, more so for historians of philosophy and logic.

As with the particular form of regimented debate, the origin of the method is difficult to ascertain. Plato described Zeno in *Phaedrus* 261d as the 'Eleatic Palamedes', i.e., as

[...] such an artful speaker that his listeners will perceive

the same things to be both similar and dissimilar, both one and many, both at rest and also in motion.

In line with the common usage, Plato called this 'antilogic'. Yet he was eager to stress that 'antilogic' should not only apply to long speeches in front of an audience, but also to any sort of speech. He presented the method as Zeno's in the *Parmenides*, both at the beginning of the dialogue and in an important methodological section which we will quote from in a moment. Aristotle is also reported to have called Zeno "the inventor of dialectics".³⁴ These are clear indications that Zeno was a practitioner of dialectical games and followed the method of antilogic. We will outline a demonstration of this below. But is he the originator of the method?

There is, of course, no basis for the claim that Zeno's teacher, Parmenides was himself a practitioner. We certainly make no such claim, although we cannot exclude it, but merely note that the fact that he wrote a hexameter poem expressing his positions does not exclude the possibility that he may have arrived at them through antilogic (the contrary claim would be a *non sequitur*). Some of the features of his *Poem* actually go a long way toward explaining the particular features of the method of antilogic.

The section of Parmenides' *Poem* we are interested in follows immediately after its introduction, known as the 'proem'. It is composed of fragments B2, B6, and B7 along with lines 1–2 and 15–16 of b8:

B2. Come now, and I will tell you [...] the only ways of enquiry that are to be thought of. The one, that [it] is and that it is impossible for [it] not to be, is the path of Persuasion (for she attends upon Truth); the other, that [it] is not and that it is needful that [it] not be, that I declare to you is an altogether indiscernible track: for you could not know what is not—that cannot be done—nor indicate it.

B6. What is there to be said and thought must needs be: for it is there for being, but nothing is not. I bid you ponder that, for this is the first way of enquiry from which I hold you back, but then from that on which mortals wander knowing nothing, two-headed, for helplessness guides the wandering thought in their breasts, and they are car-

ried along, deaf and blind at once, dazed, indiscriminating hordes, who believe that to be and not to be are the same and not the same; and the path taken by them all is backward turning (*palintropos*).

B7. For never shall this forcibly be maintained, that things that are not are, but you must hold back your thought from this way of enquiry, nor let habit, born of much experience, force you down this way, by making you use an aimless eye or an ear and a tongue full of meaningless sound: judge by reason the strife-encompassed trial (*poluderis elenchus*) spoken by me.

B8. There still remains just one account of a way, that it is.
[...] And the decision about these things lies in this: it is or it is not.³⁵

One should note the mention of the *poluderis elenchus*, i.e., at the end of B7. It implies that Parmenides saw the content of these passages not as exhibiting a ‘refutation’, as this might be an incorrect translation of *elenchus*, but rather a *trial* or a *test*. Parmenides is discussing here possible routes or ways of enquiry, that start either with ‘it is’ (*estin*), ‘it is not’ (*ouk estin*), or a combination of both, suggested in B6: “to be and not to be are the same and not the same”.³⁶ It is these that have to be tested.

We are interested in the deductive structure of his argument, in the form of a double disjunctive syllogism with the premise:

$$A \vee \neg A \vee (A \& \neg A)$$

This has been denied, but at an exorbitant price, that of affirming that Parmenides had worse logical skills than Sextus Empiricus’ dog.³⁷ On the other hand, it must be said that accounts of the possible antilogical structure of the *Poem* are, so far, not very sophisticated.³⁸

In the briefest possible terms, Parmenides reasons as follows. One of his tasks is to block the third way, ‘A & $\neg A$ ’. In essence, his argument is that ‘A & $\neg A$ ’ is contradictory: the path is ‘*palintropos*’ or ‘backward turning’. So one is left with the other two alternatives, and in B8 the

point is clearly expressed: “the decision about these things lies in this: it is or it is not”. This is often read as the Law of Excluded Middle, but Parmenides is asking for a choice between A and $\neg A$, having eliminated a combination of both as the third alternative. It is worth recalling here the definition of exclusive disjunction:

$$(A \vee B) \& \neg (A \& B)$$

and to notice that Parmenides is in effect arguing that, the choice between A and $\neg A$ has to be exclusive. His other task is to block the second way, which he does in B2 with a short argument: “for you could not know what is not—that cannot be done—nor indicate it”. The point seems to be that one cannot even begin travelling down that path, because it is ‘indiscernible’.³⁹ Finally, the beginning of B8 simply affirms the result of a double application of the disjunctive syllogism to the starting premise: “There still remains just one account of a way, that it is”.

To see what possible connection this argument might have with antilogic, we have to keep in mind that Parmenides has been talking about submitting to a test three ways of enquiry: A, $\neg A$, and ‘A & $\neg A$ ’, all of which are incompatible with one another. And it is clear at least that the third way is rejected because it immediately leads to contradiction.⁴⁰ This suggests that one way to test both A and $\neg A$ is to look for contradictions on both sides. This is not exactly Parmenides stand-point, given that he argues for no contradiction on one side, namely that of ‘it is’ (*estin*) taken *simpliciter* as ‘it is there for being’ (*esti einai*, B6), but this is enough to get us going.

Before moving on to Zeno, we would like to point out another characteristic of the *Poem* that arguably has its roots in antilogic. If one is to look for contradictions, so simple an assertion as ‘it is’, especially since it is not paradoxical or self-refuting, will not be enough on its own. One needs to appeal to a set of predicates to try and engender contradictions, i.e., to travel down that path, as Parmenides would say. These are part/whole, limited/unlimited, etc. As we shall see, Parmenides’ carefully chosen set of predicates in B8 is also to be found in Zeno, Gorgias, and Plato.

For our brief presentation of Zeno’s arguments about divisibility, we chose as our point of departure Plato’s *Parmenides*. That his remarks on Zeno may be taken as reliable has been controverted, and

it is rather certain that the meeting between Parmenides, Zeno, and Socrates is imaginary, but it seems equally wrong to think that, given Plato's limited audience in his own time, he could have written anything fanciful about Zeno. In the opening pages, there is a precious exchange between young Socrates and Zeno, concerning Zeno's book, which is reported here to have been written in defence of the monism of his master, Parmenides, against the contradictory thesis that 'it is many' (*esti ta polla*). Socrates first asks:

Is this the point of your arguments—simply to maintain, in opposition to everything that is commonly said, that things are not many? And do you suppose that each of your arguments is proof for this position, so that you think you give as many proofs that things are not many as your book has arguments?⁴¹ Is that what you're saying—or do I misunderstand? (127e)

To which Zeno replied:

No [...]. On the contrary, you grasp the general point of the book splendidly. (127e–128a)

One might infer from this that Zeno is, in the fashion of Vlastos' 'Socratic *elenchus*' discussed above, providing numerous instances of *reductio ad absurdum* for the thesis 'it is many', thus proving as many times that 'it is one'. But Zeno adds a little bit further on in the conversation:

Still, you haven't completely discerned the truth about my book [...]. The truth is that the book comes to the defence of Parmenides' argument against those who try to make fun of it by claiming that, if it is one, the consequence for the argument is to suffer many things both ridiculous and contradictory to it (*polla kai geloia sumbainei paskhein tōi logōi kai enantia hautōi*). Accordingly my book contradicts (*antilegei*) those who assert the many and pays them back in kind with something for good measure. Since it aims to make clear that their hypothesis, if it is many, would, if someone examined the matter thoroughly, suffer consequences even more ridiculous than those suffered by the hypothesis if its being one. (128b–d)⁴²

Zeno's revised claim here is that he merely drew contradictions from the opposite thesis in order to contradict it, and this is in line with the conception of antilogic developed in this paper. It is interesting to note that Zeno says here that he is giving a reply to critics of Parmenides who already argued in the same manner as he did. This 'game of contradicting' had probably been going on since Parmenides, who could very well be arguing *contra* a rival school.

Zeno's four arguments about divisibility are the only surviving set which is extensive enough for us to draw some conclusions concerning his method. Aristotle himself frequently alludes to these arguments as being 'against motion', a frequent misunderstanding.⁴³ Under a reading harking back to the opening pages of the *Parmenides*, and revived in the 19th century in Paul Tannery's seminal paper,⁴⁴ they are meant to defend Parmenides' monism by deriving contradictions from the contrary hypothesis, 'if it is many' (*ei polla esti*). It is true that the contradictions involve motion, but they are not *against* motion. To recall Plato's words in the *Phaedrus* 261d, "his listeners will perceive the same things to be [...] *both at rest and also in motion*" (our italics).

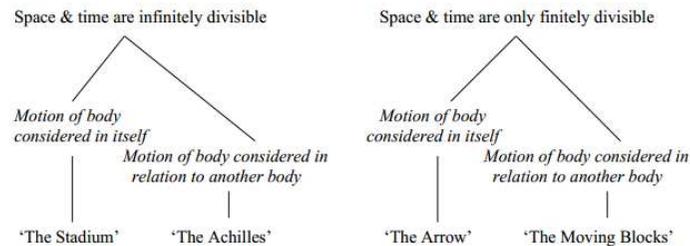
In antilogical form the arguments would (roughly) run like this. Zeno would secure agreement from his adversary that 'there is motion', a typical *endoxa* if there is one (and one that must have been brought against Parmenides, since for him Being does not undergo any change). Next, supposing that 'it is many', suppose that 'space and time are divisible' (to assume the contrary would lead one into immediate difficulties). He would then raise a question of the form 'Is it A or $\neg A$?', namely: 'Are they infinitely divisible or not?' Supposing that the adversary chooses infinite divisibility, with 'The Stadium',⁴⁵ Zeno would get his adversary to concede that the runner wishing to reach the other end of a stadium would not move, because he would have first to traverse its first half, then the first half of the first half, and so forth *ad infinitum*.⁴⁶ If, however, the adversary were to choose instead that space and time are only finitely divisible, i.e., that there are indivisible *minima*, then by 'The Arrow',⁴⁷ Zeno would get him to concede the claim that an arrow in flight must be motionless in an instant, because if it were to change position, the instant would be divisible. So the arrow is always at rest.

Thus Zeno provided two arguments: one for A (infinite divisibility)

and one for $\neg A$ (finite divisibility). As we know, however, the ‘paradoxes’ come in pairs: ‘The Moving Rows’ is, like ‘The Arrow’, also an argument deriving a contradiction from the claim that ‘space and time consist of indivisible minima’, while ‘The Achilles’ complements the ‘The Stadium’ for the opposite supposition. Why four arguments divided in pairs? The answer is in the methodological passage towards the middle of *Parmenides*, where Parmenides tells the young Socrates:

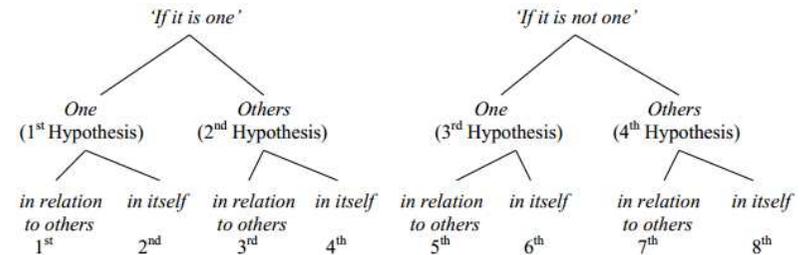
take as an example this hypothesis that Zeno entertained: if many are, what the consequences must be for the many themselves in relation to themselves and in relation to the one, and for the one in relation to itself and in relation to the many? And, in turn, on the hypothesis, if many are not, you must again examine what the consequences will be both for the one and for the many in relation to themselves and in relation to each other. (136a–b)

Using this as a cue, we can see that in each pair, one argument derives a contradiction from a body’s motion considered *in itself*, these are ‘The Arrow’ and ‘The Stadium’, while the other arguments, ‘The Achilles’ and ‘The Moving Rows’, derive a contradiction from a body’s motion *in relation to* the movement of another body. So the arguments about divisibility would be structured like this:⁴⁸



This example should illustrate the method of antilogic. Although it is not possible even to begin arguing for this point properly, one could suggest that this is also the structure of the eight series of deductions in the second half of *Parmenides* (137c–166c). Indeed, in the very same fashion, the arguments are structured around the supposition ‘If it is one’ (*ei hen estin*) and its negation, and consequences for the ‘one’

(or ‘unity’) are deduced (1st Hypothesis) that are at first negative (first series of deduction) and then positive (second series of deduction), and so forth, and organized around the distinction between ‘in itself’ and ‘in relation to others’, to give the following structure:⁴⁹



A proper understanding of the structure of these arguments is no idle task, given the historical importance of Neoplatonist readings of the dialogue, which read the first deduction of the first hypothesis as a form of negative theology meant to reveal an unknowable and ineffable God beyond reality. Such readings destroy the antilogical structure of the second part of the *Parmenides*, as well as leaving unexplained the presence of the other series of deductions.⁵⁰

The arguments of the second part of *Parmenides* also share two further features with Parmenides’ poem. First, there is the odd case of a third series of deductions under the 1st hypothesis, at 155e–157b, of the form ‘A & $\neg A$ ’:

If the one is [...] both one and many and neither one nor many. (155e)

This is reminiscent of Parmenides’ third way. As a matter of fact, the first hypothesis as a whole thus looks very ‘Parmenidian’, since it investigates in turn, A, $\neg A$ and ‘A & $\neg A$ ’. This is also the structure of Gorgias’ ontological argument at the beginning of his treatise *On Not-Being*,⁵¹ since he derives contradictions in every branch of this hypothesis:

If anything exists either it is the existent that exists or the non-existent, or both the existent and the non-existent exist.⁵²

Likewise, Gorgias goes on deriving contradictions for the three paths, blocking them all.

This feature, typical of Eleatic antilogic, has disappeared in Aristotle, who has bouts beginning with problems of the form ‘Is it A or $\neg A$?’ (*Topics* I, 4, 101^b32–33 and VII, 2, 158^b14–25). For example, Aristotle in *De Caelo*, I, 10, 297^b4 f., the question is ‘Is the world eternal or not?’. It is an important point to make in relation to the interpretation of Aristotle that he kept the idea of an enquiry that would *first* proceed by deriving contradictions on both sides of a problem and frequently speaks in this context of *aporia*—‘difficulties’ or ‘puzzles’—as in *Nicomachean Ethics* VII, 1145^b2–7:

We must, as in all other cases, set out the phenomena before us and, after first [going through all the] difficulties, go on to prove, if possible, the truth of all the reputable opinions about these affections or, failing this, of the greater number and the most authoritative; for if we both resolve the difficulties and leave the reputable opinions undisturbed, we shall have proved the case sufficiently.⁵³

Aristotle further points out in *Topics*, VI, 6, 145^b17–20 that *aporiai* emerge precisely when, ‘going through the difficulties on either side’, the results turn out to be equally puzzling:

Likewise also an equality between contrary reasonings would seem to be a cause of perplexity; for it is when we reflect on both sides of a question and find everything alike to be in keeping with either course that we are [puzzled] about which one we are to do. (Translation slightly modified.)

The most significant example of this is *Metaphysics* Book III, which contains a discussion of 14 such *aporiai*, about which Aristotle simply does not tell us which side he takes.⁵⁴ That Aristotle believed that this method of antilogic, i.e., ‘going through the difficulties on either side’ is a *necessary* condition for philosophy, is expressed at *Topics*, VIII, 14, 163^b9–16:

[...] when it comes to knowledge and the wisdom that comes from philosophy, being able to discern—or having already discerned—the consequences of either assumption

is no small instrument: for it remains to choose one or the other of these rightly. In order to do that, one must be naturally gifted with respect to truth: to be properly able to choose the true and avoid the false. This is just what the naturally good are able to do, for it is by loving and hating in the right way whatever is presented to them that they judge well what is best.

There is an exact correspondence with Aristotle’s own practice, as he begins his treatises in this way.⁵⁵

One final point about predicates. We noted earlier that, suppositions such as ‘it is one’ not being evidently self-contradictory, one needs, in order to derive contradictions, to consider the bearing of the suppositions on a set of qualifications, such as limited/unlimited, motion/rest, equal/unequal, etc. As Reginald Allen once noticed, the list of the qualifications in the *Parmenides* “reflects the Eleatic tradition”, as the list is almost fully represented in Parmenides’ *Poem* (B8), and “may indeed conform to the principal divisions of Zeno’s book” (Allen 1983, 188 & 306-307n. 80). If we also list predicates involved in Gorgias’ ontological argument, we find a remarkable overlap, even though we do not have, in the case of Zeno, a complete set of arguments. The following table, from a partial list, should suffice to illustrate our point:

	Limited/ Unlimited	In itself/ In another	Divisible/ Indivisible	Like/ Unlike	Contact/ No contact	Part/ Whole	Equal/ Non equal	Motion /Rest
Parmenides	x	x	x	x	x	x	x	
Zeno	x	x	x	x	x			x
Gorgias	x	x	x					x
Plato	x	x	x	x		x	x	x

Aristotle also proposed a list of ‘categories’ in *Topics* I 9, 103^b20–26:

Now, then, next after this we must distinguish the categories of predications in which the four <types of> predication mentioned are found. These are ten in number: what-it-is, quantity, quality, relation, location, time, position, possession, doing, undergoing. An accident, a genus, a unique property, and a definition will always be in one of these categories, for all the premises <produced> by means of them signify either a what-it-is, or a quantity, or a quality, or some one of the other categories.

This list is of course very closely related to the ten-fold division in the *Categories* (there is only a difference in the name of the first category). In his notes to the *Categories*, J. L. Ackrill suggested that Aristotle arrived at his list of categories by reflecting on questions, e.g., noticing that

An answer to ‘where?’ could not serve as an answer to ‘when?’. (Ackrill 1963, 78)

This view has been debated since by commentators, but we feel it must be on the right track: our suggestion would be that Aristotle’s list originates in the practice of question and answer. There are some major differences, however, between Aristotle and his predecessors in this respect. While they used a specific set of contradictory predicates that had been carefully selected originally by Parmenides for his own purposes, Aristotle was far more encompassing and systematic in his classification of predicates of Greek language. Furthermore, Aristotle’s classification is not in terms of pairs of contradictory predicates. Nevertheless, the presence of the list of categories in *Topics* I 9 suggests that his systematic study was for the purpose of advancing knowledge of the predicates involved in antilogic. After all, *Topics* is a handbook to teach players how to excel in the game, and categories are involved in an essential way in the *topoi*.

3. CONCLUDING REMARKS: THE ORIGIN OF LOGIC

Logic is reputed to have originated in Aristotle’s syllogistic and in the writings of Stoics such as Chrysippus. The study of logic in Ancient philosophy was renewed in the last century following the advent of modern logic, in particular by Jan Łukasiewicz, but in his study of *Prior Analytics*,⁵⁶ he did not take antilogic into consideration, and he forced upon the text a particular syntactic approach, that of axiomatics, that clearly did not fit it; he ended up claiming that syllogisms are not valid inferences but true universalized conditionals, and portrayed syllogistic not as the underlying logic of the theory of demonstrations in *Posterior Analytics*, but as a logic of terms that presupposed in turn a propositional logic as underlying logic.⁵⁷ He thus faulted Aristotle for having ignored the latter, and rightly credited the Stoics for its invention. Under those conditions, it is difficult to see why Aristotle should

be considered the inventor of logic, as we understand the term, if only because of, say, his use of schematic letters.⁵⁸

This is not the place for a critique of this approach, already severely undermined since pioneering work by Timothy Smiley and John Corcoran in the 1970s.⁵⁹ We would like simply to say a word about Aristotle’s context. Much of the work in the history of logic in this tradition has shown a remarkable lack of sensitivity to it. With Łukasiewicz, history of logic looked more like an attempt at fitting Aristotle’s text onto the Procrustean bed of one’s own axiomatic conception of logic, and, although work by Smiley and Corcoran certainly fitted the text much better, it was also ahistorical. We simply submit that antilogic was an essential part of the context within which both Aristotle and the Stoics introduced their ideas. The guiding ideas of game semantics allowed us to reconstruct antilogic, and rules 1–5 indicate that bouts proceeded in two basic steps: first, in a series of questions and answers, *Questioner* was able to elicit agreement from *Answerer* to the theses that entail the contradictory of the thesis with which the bout started, and, secondly, from these *Questioner* inferred the impossibility (*adunaton*). The logical steps at the end of the bouts were at first not made explicit in terms of rules of inference because of their intuitive obviousness, but it is not surprising that Aristotle, and later on the Stoics, would search for rules here, so as to teach their players which premises they needed to perform in order to undergo an *elenchus* successfully.⁶⁰ Now what was thus *made explicit*, we have not inquired into yet. This is a task for logicians and linguists, and we merely propose the foregoing as the basis for this investigation. Antilogic was an essentially *enactive* conception of logic, where the role of the second player was crucial, something only game semantics allows us to recover. Given the importance of antilogic for philosophical enquiry, playing was more important than metatheoretic reflection, but that should not mean that the likes of Zeno, Gorgias and Plato should not count among the most remarkable logicians in history.

Notes

¹The expression ‘Socratic *elenchus*’ would lead one to believe that either Socrates invented the method or that no one used it after him or both. These claims, however, can only be upheld by blatantly disregarding a large amount of evidence pointing to widespread use of that word prior to and after Socrates. Terminological issues are also

complicated by the fact that the words ‘*dialektike*’ and ‘*eristike*’ were at first used to describe the same method, but the latter took on negative connotations, often associated with the Sophists, whose name became synonymous with misuses of the method, for which we now have the word ‘sophistry’, while the former came to acquire a variety of meanings that have nothing to do with the ‘method of the elenchus’. But these points should not detain us. See Nehamas (1990) for a discussion on ‘dialectic’, ‘eristic’ and ‘antilogic’ in Plato.

²On the Nature of Man, I. Translation from Jones (1947), modified.

³Previous work in the wake of Lorenzen, includes Ebbinghaus (1964); Krabbe (2006); Castelnérac & Marion (2009); Rahman & Keiff (2010); Marion & Rückert (unpublished). Our reason for eschewing any reference to dialogical logic is to keep our presentation at the most general level, unrestricted by any specific ties to the latter.

⁴A possible line of inquiry that we have in mind is the study of Plato’s dialogues, as illustrations of dialectical games, from the standpoint of ludics. Alain Lecomte and Myriam Quatrini have applied ludics to the study of dialogue as a game; see, e.g., Lecomte & Quatrini (2011), and Part IV of Lecomte (2011). This could usefully be applied to the study of Plato’s dialogues, to learn more about the way they function—one of us is currently working with Alain Lecomte in applying this setting to the study of Lesser Hippias.

⁵Also called ‘dialectical games’ in Castelnérac & Marion (2009).

⁶Another reason is the very nature of dialectical games, as any report of these verbal encounters would have been from memory. Furthermore, there was no literary style that would have consisted of straightforward reports of particular dialectical bouts. The closest we have to this is one short argument in Aristophanes’ *Clouds*, some pages in Xenophon, and the highly elaborate works of Plato. Worse, famed practitioners such as Socrates or Arcesilaus would not commit anything to writing. Therefore, arguments with a dialectic origin were for the most part written down in a form that was not ‘dialogical’. Some arguments, such as the Sorites, clearly retained their dialectical form, others did not and one has to use care in reconstructing them. We shall see an example of the issues involved below with Zeno’s arguments about divisibility.

⁷Diogenes Laertius, *Lives of Eminent Philosophers*, IX, 8, 55. Here ‘opposing arguments’ translates ‘antilogion’

⁸Aristotle wanted also to contrast, at the end of *On Sophistical Refutations*, his own *Topics* with the unsystematic teachings of the Sophists. The latter is organized in eight books, with the first and last ones providing a description of the games and their rules, along with some advice to the players, while the intervening 6 books are a catalogue of more than 300 *topoi*, i.e., ‘places’ or ‘locations’, the purpose of which is, as Jacques Brunschwig so aptly put it, “to frame premises from a given conclusion” (Brunschwig 1967, xxxix). For example, given that the answerer asserts *A*, the questioner must then drive him into a contradiction, i.e., to find ways of having him concede $\neg A$, and the appropriate *topos* is meant to help the questioner find the further premises needed to entail $\neg A$, so that he has to first get the proponent to concede these, and only then to draw the inference to the contradiction. All of this is lacking in texts such as *Dissoi Logoi*, hence Aristotle’s pride at 34, 183^b36 & 184^b1 that nothing of the sort existed before him. In this respect, it is quite interesting to note that in this last passage, he also claims that his work is the result of “experimental researches”; this implies having played an incredible amount of games.

⁹In the very same passage Aristotle mentions his “present inquiry” and this is meant to cover both books, so *On Sophistical Refutations* should simply be seen as the ninth and final book of *Topics*.

¹⁰See Dillon & Gergel 2003, 320–333).

¹¹Dupréel (1922); Dupréel (1948) thought the resemblance between the *Dissoi logoi* and Plato’s dialogue so conspicuous that he accused the latter of simply plagiarizing the sophists; of course, this is going too far, but it is true that Plato is not purely original in the topics he chose and the way his characters argue against one another.

¹²The contrast is also drawn with other prominent Sophists: Hippias in *Lesser Hippias*, 373a, and Protagoras in *Protagoras* 334c-d. In both cases, a wily Socrates invokes his lack of memory, but the most important reason he invokes is that speeches, like books, do not answer questions, at *Protagoras*, 329a-b. (This is, of course, in line with Socrates’ notorious critique of writing at *Phaedrus* 276a-d.) This contrast should not mislead one into thinking that the Sophists were not skilled in the art of dialectical games, as he portrays both Protagoras and Gorgias as capable of playing them at, respectively *Protagoras*, 335a and *Gorgias*, 449b-c, quoted here. Given what we know of them, and their popularity when they were alive, to claim the contrary would have been odd, especially in his days. Gorgias’ treatise *On Not-Being* is certainly a dialectical *tour de force*.

¹³From which are derived the modern words ‘macrology’ and ‘brachylogy’, with slightly different connotations.

¹⁴As it happens, Plato’s dialogues do not merely contain ‘yes/no interrogatives’, as ‘wh-interrogatives’ also frequently occur. Aristotle, however, insisted in this passage that dialectical games should contain only ‘yes/no interrogatives’ and showed how to transform some ‘wh-interrogatives’ into ‘yes/no interrogatives’: instead of asking ‘What is the *F*’, e.g., ‘What is courage?’, one simply has to ask a disjunction ‘Is courage *X* or not *X*?’

¹⁵For a full analysis, see Brisson (1984).

¹⁶Still, this is a historically important text, given that it is a key to Neoplatonism, and that the latter played an important role in the reception of Plato from the Renaissance onwards. It is fitting to note that the modern commentator closest to Neoplatonic readings of the second part of *Parmenides*, F. M. Cornford also translated it as a simple monologue, factoring away the young Aristotle’s answers, thereby erasing the dialectical dimension and what it may bring to the understanding of the text (Cornford 1939).

¹⁷The notion of ‘scoreboard’ comes from Lewis (1983) essay 13. One could alternatively speak of a ‘commitment store’, following Hamblin (1970).

¹⁸See Crubellier 2011, 25–26.

¹⁹See Lear 1980, 8. For more on the intuitionist notion of ‘interpretation by replacement’, see Dummett 1977, 218f.

²⁰It would be interesting to relate this approach, based on an intuitive recognition of the validity of some basic cases, to John Etchemendy’s notorious critique in Etchemendy (1990) of the model-theoretical account of logical consequence as being unable to capture the intuitive notion of the validity of an inference.

²¹The argument has a strong realist flavour: anyone rejecting the Principle of Non Contradiction and who entertains contradictory beliefs would amount to a self-contradictory object, as these contradictory beliefs would be contradictory attributes of the same subject, and this cannot be the case. See Evans 1999, 139.

²²DK 82 B 11, §25. Translation in Dillon & Gergel 2003, 90–91.

²³See, e.g., Benson (1995). See also, below, the discussion of Zeno in the opening

pages of *Parmenides*.

²⁴Donald Davidson, who saw here connections with his own ideas in ‘A Coherence Theory of Truth and Knowledge’ (Davidson 2001), pointed out that, according to Vlastos, Plato must have realized “what must be assumed if the elenchus is to produce truths: the assumption is that, in moral matters, everyone has true beliefs which he cannot abandon and which entail the negations of his false beliefs” (Davidson 2005, 229). Thus Socrates was, according to Davidson, in possession of a “method that leads to truth” (Davidson 2005, 239), throughout gradual elimination of false beliefs. It seems to us that the same objection can be made here, given that the point of dialectical games is to test the consistency of sets of beliefs: there is never a guarantee that one has eliminated false beliefs rather than true ones. It is always possible that the consistent set one ends up with contains only false beliefs, so the possibility of massive error remains open.

²⁵See Vlastos 1994, 13–17). For more details see Castelnérac & Marion 2009, 57–59. For Socrates’ avowals of ignorance, see Castelnérac & Marion 2009, 61–62).

²⁶The Formal Rule has no parallel in other forms of game semantics. See Castelnérac & Marion 2009, 61 and section 5 of Marion & Rückert (unpublished) for detailed explanations.

²⁷The other passage is at *Sophistical Refutations*, 34, 185a25.

²⁸See endnote 4 above.

²⁹The rule is also stated at *Topics*, VIII, 8, 160b1–6. It is also related to the rule for universal quantification in dialogical logic, see section 6 in Marion & Rückert (unpublished) for a detailed discussion.

³⁰Aristotle even attributes to Socrates the invention of inductive arguments (*Metaphysics*, XIII, 4, 1078^b 28). This was common knowledge in the 19th century, and thus far the only logical skill broadly attributed to Socrates.

³¹For a detailed discussion of these last points, see Marion & Rückert (unpublished).

³²This is what we called the ‘method of dialectical games’ in Marion & Rückert (unpublished).

³³Other arguments by Zeno and further arguments by Melissus were probably of this nature, but the little that we have in our possession is too fragmentary for a reconstruction. For an example of an antilogical reconstruction of an argument by Melissus, see Makin (2005).

³⁴Sextus Empiricus, *Against Logicians*, I, 7, and Diogenes Laertius, *Lives of Eminent Philosophers*, IX, 25.

³⁵Respectively, §§291, 293, 24 and 295 in Kirk et al. (1983), modified.

³⁶Not all commentators agree as to the existence of this third way, see, e.g., the arguments in Curd 2004, 51–63).

³⁷Sextus Empiricus, *Outline of Pyrrhonism*, I, 69. For this particular claim, see Leshner (1984), 13. In all fairness to Leshner, such odd claims about the structure of the argument have practically disappeared from the revised (Leshner 2002).

³⁸The *locus classicus* here remains Montgomery Furth’s claim that “the essence of Parmenides’s procedure [is] that he is not at this point putting forth an ontology of his own, but is practicing dialectical criticism” (Furth 1968, 118).

³⁹On a reading that takes ‘*estin*’ as both existential and predicative, that this second way is ‘indiscernible’ may mean that no clear thought can be expressed by a negative existential proposition.

⁴⁰Parmenides is no ‘dialetheist’ in the sense of Priest (2002).

⁴¹Proclus, possibly one of the last persons to have seen it, reported that it contained 40 *logoi* in his *Commentary on Plato’s Parmenides*, 694, 23 f. See Morrow & Dillon (1987), 72.

⁴²Translation slightly modified.

⁴³For example, in *Prior Analytics* B 17, 65^b 18–19 or *Topics*, Θ, 8 160^b 8–9.

⁴⁴See Tannery (1885).

⁴⁵At least under the version at *Physics* Z 9, 239^b 11.

⁴⁶The version reported at *Physics*, Θ 8, 263^a 15–18, 263^b 3–9 has it that a runner could not reach the other end of a stadium because he would have to traverse an infinity of points in a finite time, which is impossible, although a runner can reach the other end of a stadium.

⁴⁷See *Physics* Z 9, 239^b 5–7.

⁴⁸This reading is hardly new, it was suggested by the historian of Greek mathematics, Sir Thomas Heath in Heath 1921, vol. 1, 273–283), and developed since by G. E. L. Owen in ‘Zeno and the Mathematicians’ (Owen 1986).

⁴⁹This reconstruction is partly based on Gill & Ryan 1996, 57–58 and Brisson 1999, 46, and needs to be confirmed by a detailed exegesis. This approach depends on taking the phrases ‘*pros heauto*’ and ‘*pros ta alla*’, i.e., ‘in relation to itself’ and ‘in relation to the others’ to have a non-technical, straightforward meaning, and differs in this from Meinwald 1991, chap 3).

⁵⁰For a critique see Dodds (1928) and Allen 1983, 189–195.

⁵¹Of which there are two versions: Sextus Empiricus’ *Against Logicians* VII, 65f. and Pseudo-Aristotle’s *On Melissus, Xenophanes and Gorgias*, 979^a 12 f. For a comparative study of the structure of Gorgias’ argument in both versions, see Castelnérac (forth.).

⁵²We quote here from Sextus Empiricus, *Against Logicians*, VII, 65, translation in Dillon & Gergel 2003, 68.

⁵³Translation slightly modified.

⁵⁴See Crubellier & Laks (2009).

⁵⁵For this point, see, e.g., Brunschwig 1967, xvi–xvii) or Smith 1997, xvii–xix).

⁵⁶See Łukasiewicz (1957).

⁵⁷See Łukasiewicz (1968).

⁵⁸On this point, see, *inter alia*, Corcoran (1974b) and Corcoran (2009).

⁵⁹The key papers here are Smiley (1973) and Corcoran (1974a). See also Lear (1980) and Smith (1989), which incorporates the Smiley-Corcoran approach in the translation and commentary.

⁶⁰This idea was first expressed in Hintikka (1993). See also Hintikka 2007, 3, Castelnérac & Marion 2009, 78 and Crubellier 2011, 25.

References

We use throughout standard conventions for referring to the texts, and we have used translations in easily available editions when possible, namely (Kirk, Raven & Schofield 1983) for Parmenides and (Dillon & Gergel 2003) for Gorgias, while translations from Plato are from (Cooper 1997), except for Parmenides, for which we used (Gill & Ryan 1996), and translations from Aristotle from (Barnes 1984), with the exception of (Smith 1989) for *Prior Analytics* and (Smith 1997) for *Topics*, I and VIII. Otherwise, the refer-

ence is provided in the footnote.

- Ackrill, John L. (ed.). 1963. *Aristotle: Categories and de Interpretatione*. Oxford University Press.
- Allen, Reginald E. (ed.). 1983. *Plato's Parmenides*. Minneapolis, University of Minnesota Press.
- Barnes, Jonathan (ed.). 1984. *The Complete Works of Aristotle: The Revised Oxford Translation*. Princeton, Princeton University Press.
- Benson, Hugh H. 1995. 'The dissolution of the problem of the elenchus'. *Oxford Studies in Ancient Philosophy* 13: 45–112.
- Brisson, Luc. 1984. 'Les réponses du jeune Aristote dans la seconde partie du Parménide de Platon. Essai de classification'. *Revue informatique et statistique dans les sciences humaines* 20: 59–79.
- Brisson, Luc (ed.). 1999. *Platon. Parménide*. Princeton, Paris, Flammarion.
- Brunschwig, Jacques. 1967. *Aristote: Topiques Livres I-IV*. Paris, Belles Lettres.
- Castelnérac, Benoît. forth. 'L'argument ontologique dans le traité Sur le non-être de Gorgias'. *Phoenix*, to appear.
- Castelnérac, Benoît & Marion, Mathieu. 2009. 'Arguing for inconsistency: dialectical games in the academy'. In G. Primiero & S. Rahman (eds.) 'Acts of Knowledge: History, Philosophy and Logic', 37–76. London: College Publications.
- Cooper, John M. (ed.). 1997. *Plato: Complete Works*. Indianapolis: Hackett.
- Corcoran, John. 1974a. 'Aristotle's natural deduction system'. In J. Corcoran (ed.) 'Ancient logic and its modern interpretations', 85–131. Dordrecht, D. Reidel.
- . 1974b. 'Aristotelian syllogisms: valid arguments or true universalized conditionals?' *Mind* 83, no. 330: 278–281.
- . 2009. 'The Founding of Logic. Modern Interpretations of Aristotle's Logic'. *Ancient Philosophy* 14: 9–24.
- Cornford, F. M. 1939. *Plato and Parmenides. Parmenides' Way of Truth and Plato's Parmenides*. London, Routledge & Kegan Paul.
- Crubellier, Michel. 2011. 'Du *sullogismos* au syllogisme'. *Revue philosophique* 1/2011: 17–36.
- Crubellier, Michel & Laks, André. 2009. *Aristotle's Metaphysics Beta: Symposium Aristotelicum*. Oxford University Press.
- Curd, Patricia. 2004. *The Legacy of Parmenides*. Las Vegas, Parmenides Publishing.
- Davidson, Donald. 2001. 'A coherence theory of truth and knowledge'. In 'Subjective, Intersubjective, Objective', 137–153. Oxford, Clarendon Press.
- . 2005. 'Plato's philosopher'. In 'Truth, Language, and History', 223–240. Oxford, Clarendon Press.
- Dillon, John & Gergel, Tania (eds.). 2003. *The Greek Sophists*. London, Penguin.
- Dodds, Eric R. 1928. 'The Parmenides of Plato and the Origin of the Neoplatonic "One"'. *The Classical Quarterly* 22, no. 3/4: 129–142.
- Dummett, Michael A. E. 1977. 'Elements of intuitionism'.
- Dupréel, Eugène. 1922. *La légende socratique et les sources de Platon*. Bruxelles, Robert Sand.
- Dupréel, Eugène. 1948. *Les sophistes*. Neuchâtel, Le griffon.
- Ebbinghaus, Kurt. 1964. *Ein formales Modell der Syllogistik des Aristoteles*. Vandenhoeck & Reprecht.

- Etchemendy, John. 1990. *The concept of logical consequence*. Stanford CA, CSLI.
- Evans, J. D. G. 1999. 'Dialectic, Contradiction, and Paraconsistency'. In May Sim (ed.) 'From Puzzles to Principles? Essays on Aristotle's Dialectic', 137–149. Lexington Books.
- Furth, Montgomery. 1968. 'Elements of Eleatic ontology'. *Journal of the History of Philosophy* 6, no. 2: 11–132.
- Gill, Mary Louise & Ryan, Paul (eds.). 1996. *Plato. Parmenides*. Hackett Publishing.
- Hamblin, Charles L. 1970. *Fallacies*. Newport News VA., Vale Press.
- Heath, Thomas. 1921. *A history of Greek mathematics*, vol. 1 and 2. Oxford, Clarendon Press.
- Hintikka, Jaakko. 1993. 'Socratic questioning, logic and rhetoric'. *Revue internationale de philosophie* 47, no. 184: 5–30.
- . 2007. *Socratic Epistemology: Explorations of knowledge-seeking by questioning*. Cambridge University Press.
- Jones, W. H. S. 1947. *Hippocrates, Vol. IV*. Cambridge MA., Harvard University Press.
- Kirk, G. S., Raven, J. E. & Schofield, M. (eds.). 1983. *The Presocratic Philosophers: A Critical History with a Selection of Texts*. Cambridge, Cambridge University Press. 2nd edition.
- Krabbe, Erik. 2006. 'Logic and games'. In Peter Houtlosser & Agnes van Rees (eds.) 'Considering pragma-dialectics', 185–198. Routledge.
- Lear, Jonathan. 1980. *Aristotle and logical theory*. Cambridge, Cambridge University Press.
- Lecomte, Alain. 2011. *Meaning, logic and ludics*. London, Imperial College Press.
- Lecomte, Alain & Quatrini, Myriam. 2011. 'Figures of dialogue: a view from Ludics'. *Synthese* 183, no. 1: 59–85.
- Leshner, James H. 1984. 'Parmenides' critique of thinking. The *Poluderis Elenchos* of Fragment 7'. *Oxford Studies in Ancient Philosophy* 2: 1–30.
- . 2002. 'Parmenidean elenchus'. In Gary Alan Scott (ed.) 'Does Socrates Have a Method: Rethinking the Elenchus in Plato's Dialogues and Beyond', 19–35. Penn State Press.
- Lewis, David. 1983. *Philosophical papers, Volume I*. New York: Oxford University.
- Lorenzen, Paul. 1960. 'Logik und agon'. *Atti Congr. Internat. di Filosofia* 4: 187–194.
- Lukasiewicz, Jan. 1957. *Aristotle's syllogistic from the standpoint of modern formal logic, 2nd ed.* Oxford, Clarendon Press.
- . 1968. 'On the History of the Logic of Propositions'. In S. McCall (ed.) 'Polish Logic 1920-1939', 66–87. Oxford, Clarendon Press.
- Makin, Stephen. 2005. 'Melissus and his opponents: The argument of DK 30 B 8'. *Phronesis* 50: 263–288.
- Marion, Mathieu & Rückert, Helge. unpublished. 'Aristotle on Universal Quantification: A Study from the Perspective of Game Semantics'.
- Meinwald, C. C. (ed.). 1991. *Plato's Parmenides*. Oxford, Oxford University Press.
- Morrow, Glenn R. & Dillon, John. 1987. *Proclus' Commentary on Plato's Parmenides*. Princeton University Press.
- Nehamas, Alexander. 1990. 'Eristic, antilogic, sophistic, dialectic: Plato's demarcation of philosophy from sophistry'. *History of Philosophy Quarterly* 7, no. 1: 3–16.
- Owen, Gwilym Ellis Lane. 1986. 'Zeno and the Mathematicians'. In M. Nussbaum (ed.) 'Logic, science and dialectic: collected papers in Greek philosophy', 45–61. Duckworth.

- Penner, Terry. 2007. 'The death of the so-called Socratic elenchus'. In M. Erler & L. Brisson (eds.) 'Gorgias-Menon. Selected Papers of the Seventh Symposium Platonicum', 3–19. Sankt Augustin, Academia Verlag.
- Priest, Graham. 2002. *Beyond the Limits of Thought*. Oxford, Clarendon Press.
- Rahman, Shahid & Keiff, Laurent. 2010. 'La Dialectique entre Logique et Rhétorique.' *Revue de Métaphysique et de Morale* 66: 149–178.
- Smiley, Timothy. 1973. 'What is a Syllogism?' *Journal of Philosophical Logic* 2, no. 1: 136–154.
- Smith, Robin (ed.). 1989. *Aristotle: Prior Analytics*. Oxford, Clarendon Press.
- . 1997. *Aristotle, Topics: Books I and VIII*. Oxford University Press.
- Tannery, Paul. 1885. 'Le concept scientifique du continu: Zénon d'Élée et Georg Cantor'. *Revue Philosophique de la France et de l'Étranger* 20: 385–410.
- Vlastos, Gregory. 1994. *Socratic studies*. Cambridge University Press.