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ANCESTOR WORSHIP IN THE LOGIC OF GAMES HOW FOUNDATIONAL WERE ARISTOTLE'S CONTRIBUTIONS?

ABSTRACT: Notwithstanding their technical virtuosity and growing presence in mainstream thinking, game theoretic logics have attracted a sceptical question: “Granted that logic can be done game theoretically, but what would justify the idea that this is the preferred way to do it?” A recent suggestion is that at least part of the desired support might be found in the Greek dialectical writings. If so, perhaps we could say that those works possess a kind of foundational significance. The relation of *being foundational for* is interesting in its own right. In this paper, I explore its ancient applicability to relevant, paraconsistent and nonmonotonic logics, before returning to the question of its ancestral tie, or want of one, to the modern logics of games.

1. LOGIC AND GAME THEORY

Since its inception in the early 1940s (von Neumann & Morgenstern 1944),¹ the mathematical theory of games has become something of a boom industry, with a sophisticated and ever expanding literature in many areas of the physical and biological sciences, the behavioural

and social sciences, the formal and computational sciences, and various branches of philosophy.² In its appropriation by logic,³ the game theoretic orientation has two essential features. The first is that the logical particles - quantifiers for example - are specified by the rules governing how a player in a win-lose game responds to sentences in which the particle in question has a dominant occurrence, depending on which role in the game he occupies. The rules for this are widely referred to as the *logical* rules different rules for different roles. Consider, for example, the universal quantifier \forall . Its game theoretic provisions are given as follows: Let $A[x]$ be a formula, with x 's occurrence possibly free. Then when one party advances $\forall x A[x]$, the opposing party selects a constant a for x and challenges the first party to defend $A[x/a]$.

The second feature of the game theoretic approach is that the logic's metalogical properties - truth in a model, valid consequence, etc. - are game theoretically definable via the concept of a winning strategy. For example, given the axiom of choice it is provable that a first order sentence A is true in a model M in the standard truth conditional sense of Tarski just in case there is a winning strategy for the defender of A in a game $G(M)$ (Hodges 1983). The rules that generate winning strategies also include the game's organizational and attack-and-defend rules; the rules of procedure. Here, too, there are different rules for different roles. These are commonly known as the *structural* rules.

We now have a simple way of characterizing a game theoretic logic. It is a logic governed by these kinds of logical and structural rules.

2. A QUESTION AND A CHALLENGE

My project is motivated by a sceptical question posed by Wilfrid Hodges and a hopeful challenge issued by Mathieu Marion (Hodges (2004); Marion (2009)). The challenge is intended to play a role in arriving at a response to the question. So I begin with the question.

Hodges' Question: In his *Stanford Encyclopedia* entry on logic and games, Hodges writes:

In most applications of logical games, the central notion is that of a winning strategy for [the proponent]. Often these strategies (or their existence) turn out to be equiv-

alent to something of logical importance that could have been defined without using games - for example a proof. But games are felt to give a better definition because they quite literally supply some motivation: [the proponent] is trying to win. This raises a question that is not of much interest mathematically, but it should concern philosophers who use logical games. If we want [the proponent's] motivation in a game G to have any explanatory value, then we need to understand what is achieved if [the proponent] does win. In particular, we should be able to tell a realistic story of a situation in which some agent called [the proponent] is trying to do something intelligible, and doing it is the same thing as winning a game. (Hodges 2004 §2)⁴

Hodges goes on to say that to the best of his knowledge no satisfactory answer has yet been found (Hodges 2001, 2004, 2006).

I am not quite sure what question Hodges is asking. (He is certainly not asking what made it intelligible that Wellington should have been motivated to prevail against Napoleon in the Battle of Waterloo). But here is a possibility we might consider. \mathcal{L} is a formalized quantificational language and X is an arbitrarily selected logician wishing to write a semantics for \mathcal{L} . Accordingly, X sets out to define \mathcal{L} 's logical operators and, thereupon, to characterize \mathcal{L} 's further semantic properties. This is a perfectly intelligible motivation for X , and, of course, X will have some fairly clear conception of what he is about as he moves through this semantic agenda. Suppose now that X understands his own semantic behaviour as playing a win-lose game with an opponent. Notice that Hodges is not asking why a *game theoretic* logician might characterize himself in this way. A game theoretic logician would characterize himself in that way because that's what he is. That's the course he has already chosen to take. But, by construction of the present case, the subject of Hodges' sceptical question is any logician whomever. In which case, the question attains some purchase. But it gets its traction at a price. The price is that the question is not a serious one. What is the point of asking why, when semanticizing \mathcal{L} in the manner of, say, Tarski, would a logician conceive himself as playing the rules of a win-lose game? That was the last thing that Tarski took himself to be doing. Why, then, would we think that Tarski's efforts lacked an

intelligible motivation?

Perhaps I have got this all wrong. No matter; there is still a perfectly serious question occasioned by Hodges' remarks, if not formulated by them. I rather think that it is this unvoiced question that prompts Marion's challenge. It is a question that asks for a *justification* of a logician's preferences for semanticizing \mathcal{L} in the game theoretic way. From this point on, this is what I shall mean by "Hodges' question". It is also an interpretation discernible in the title of Marion's paper "Why play logic games?" Let's turn to that now (Marion 2009).

Marion's challenge: Marion writes as follows:

Lorenzen referred *en passant* to the practice of refutation or 'dialectics' in Ancient Greece as both the original motivation for the development of logic and as a source for dialogical logic. This suggestion, which looks merely like a rhetorical flourish, was not, as far as I know, followed up by the scholarly investigation that it clearly deserves ... (Marion 2009, p. 18)⁵

That these remarks should have occurred in a paper of this title clearly enough suggests that the origins of game theoretic logic might be traced to the beginnings of logic itself. The suggestion is confirmed lines later: "At all events, my point is merely to indicate that Greek dialectics already contain elements of an answer to Hodges' question" (Marion 2009, p. 19). Marion's challenge calls for the scholarship that might verify Lorenzen's conjecture, thereby enabling a scepticism-removing reply to Hodges's demand for a justification.

3. VERIFICATIONISM

Game theoretic semantics reflects a certain kind of philosophical orientation, in regard to which the name of Wittgenstein is frequently invoked. On this approach the meaning of a linguistic object - a sentence say - is determined by its use.⁶ It is constituted by a linguistic practice. There are, of course, a great many different kinds of linguistic practice. So varied are they that the concept of linguistic practice, like the concept of game, refuses to yield to an all-embracing definition.⁷ This, the general idea, is open to various adaptations. One is

to relativize the linguistic practices that fix the meaning of a sentence *S* to those procedures or methods that verify or disverify it.⁸ Further adaptations are also possible, one of which is motivated by an obvious question about verificationism: How are meaningful sentences to be accounted for when they lack a settled verification-disverification methodology? On a strict reading of verificationism, such sentences aren't meaningful after all. On a gentler and more plausible reading, we replace the alethic concepts of verification and disverification - that is, of showing-true and showing-false with the pragmatic concepts of defence and attack, and likewise replace the properties of truth and falsity with the pragmatic properties of victory (successful attack or defence) and defeat (unsuccessful attack or defence). Since perhaps the most usual way of managing our attack and defend practices is by way of conversational exchanges between the contending parties, a further refinement beckons. It is that the practices that confer meaning upon the sentences of our language have an inherently dialogical character. This, we might say, gives a reconceptualized version of the verificationist theory of meaning. It is a *dialogicized* adaptation of it, in which truth, the alethic property, drops out in favour of victory, the pragmatic property.

It is worth repeating that a major virtue of this dialogical approach to meaning is that it provides a way of preserving the spirit of verificationism without having to endure the massive semantic scepticism occasioned by a strict interpretation of it.⁹ It allows for large classes of meaningful sentences whose verification and falsification lie beyond our reach or are otherwise impossible.

We see in this the unmistakable presence of the concept of a game, or anyhow of an attack-and-defend contest,¹⁰ or *dialectic* as the ancients would say. There then is a clear intuitive sense in which a dialecticized verificationism is a semantics oriented to games. But it is not yet a game theoretic semantics in the modern sense. It would not be a game theoretic semantics unless it were powered by the logical and structural rules mentioned above.

It is not my purpose to presume for these reflections any very direct causal significance. I have no inclination to suppose that when in 1961 Henkin adapted game theory to logic that he was motivated by a verificationist semantics or that his aim was to give it a pragmatic retrofit.

Even so, it would not be wrong to note that a pragmaticized verificationism is an attractive conceptual space within which to achieve the further refinements of dialectification. Dialectification, in turn, gives rise to a good and necessary question: what are the prospects for a dialectified theory of meaning short of its attainment of a full-bore game theoretic status?

All I will say for now is that a dialecticized verificationism is indeed a natural harbour for the game theoretic mariner. Whatever else might be said I reserve for the final section of the paper.

4. DIALOGUES

Greek dialectics had a substantial history before the arrival of Aristotle.¹¹ But since it is widely accepted that Aristotle is the originator of systematic logic and its first talented metatheorist, I take the liberty of refining Lorenzen's conjecture and Marion's challenge: The beginnings of game theoretic logic are in Aristotle, and confirmation of this would provide the wherewithal to calm Hodges' concerns. Since Marion thinks that the scholarship required to achieve this confirmation has yet to be done, he also thinks that logicians of game theoretic bent have an interest in the repair of this omission.

It is important to see that the lack-of-scholarship claim is a localized regret. There is lots of scholarship about the logics of the *Prior* and *Posterior Analytics*,¹² and a good deal of it regarding Aristotle's various uncompleted forays into modal logic.¹³ But what seems missing is the same sort of attention to the "immature"¹⁴ or early logic of *Topics* and *On Sophistical Refutations*, the two places in which dialectical considerations are given a wholly central role.¹⁵ So we have a further refinement of the conjecture and the challenge: The logic of *Topics* and *On Sophistical Refutations* is the starting point of game theoretic logic, and we should do the scholarship necessary to show it.

Let me note in passing that Marion's call for scholarship not yet produced should not blind us to the existence of scholarly support for the idea that origins of logic are dialectical or, more broadly, dialogical,¹⁶ and for the related proposition that the founding conception of logic calls for an interrogational formatting of the subject. Neither should it cause us to forget that Jaakko Hintikka, a leading proponent

of such linkages is himself one of game theoretic logic's most active practitioners.¹⁷ However, as we saw, dialogue logics, interrogative logics, or even dialectical logics are not intrinsically game theoretic logics. No such logic has a game theoretic character unless it is governed by logical and structural rules of the kind described in section 1. Now that's a rather stringent condition; and it is not surprising that there would be logics of this general sort that don't fulfill it.

This gives us two options to consider. One is that in advancing his dialectical origins thesis, Hintikka does indeed invest ancient logic with game theoretic purport. In which case, Marion is wrong about the absence of scholarship in this area, and yet Hodges - and by extension, Marion too - could be right in thinking that Hintikka's rationale for preferring the game theoretic approach is unconvincing.¹⁸ If that were so, Marion's challenge could be revised: Go back to the scholarship and see if you can see in Aristotle's early writings a better rationale for game theoretic preferences than the one that Hintikka himself may have extracted from them. The second option is that Hintikka did intend to construe Aristotle's logic as a logic of games, but without giving it a game theoretic characterization in the modern sense. In which case, the rationale for game theoretic preferences which Hintikka actually gives and which Hodges resists would be separate from the rationale Marion thinks might be found in dialectic.

5. FOUNDATIONALITY

In the sections to follow I want to pause, and to reserve consideration of game theoretic matters, concentrating instead on a description of Aristotle's logic as he formulates it in these texts. In the sections after that, the game theoretic question can be re-opened and, I hope, settled. I am temporarily dropping the question of Aristotle's game theoreticity not to evade it, but rather to prepare the ground for answering it. I want to approach the question of what is to be found in Aristotle's logic unencumbered by preconceptions of what our search will reveal.

Before moving on, this would be a good place to indicate some of the questions that I am *not* asking in this paper. I am not asking whether the logic of the early writings admits of reinterpretation in a game theoretic one, either extant or purpose-built. I am not ask-

ing whether there is a generic notion of game floating about in *Topics* and *On Sophistical Refutations*. What I *am* asking is whether there is recognizable in these writings a notion of game that anticipates in an appropriately robust way the notion to which von Neumann and Morgenstern gave expression in 1944. I am asking whether the logic of these texts is of *foundational* significance for game theoretic logic. I am asking whether the notion of game theoreticity was in Aristotle's logical DNA.

It might be thought that asking this foundational question of Aristotle pretty much guarantees it a negative answer. If it did, that would not be the fault of the question. The question is the right question. If there were no plausible case for thinking that, however inchoately and tentatively, Aristotle was working a game theoretic agenda, what promise could there be for the idea that it is Aristotle's logic that grounds a satisfactory *justification* of modern game theoretic preferences?¹⁹

It is also important to stress that this interest in the ancients is not an antiquarian one. We are looking to Aristotle in hopes of making some headway with a contemporary problem of logical theory. So it is only natural that we would try to determine how good a logic Aristotle's is and whether its founder has bequeathed to his distant heirs *anything* of enduring logical value, apart from its game theoretic significance or lack of it. I daresay that this, too, will strike a good many logicians as too much to ask, a question wholly without prospect of an affirmative answer.²⁰ My opinion is that we should follow Marion's advice: Look, and then see.

6. ARISTOTLE'S EARLY LOGIC

A proper understanding of Aristotle's logic requires a distinction between in my words, not his arguments in the broad sense and arguments in the narrow sense. There is a corresponding difference between a theory of argument in the broad sense and a theory of argument in the narrow sense. Arguments in the broad sense are social events. They are structured by interactive exchanges of speech acts by two or more parties. Aristotle distinguishes four kinds of arguments in the broad sense. They are refutation arguments, instruction arguments, examination arguments and demonstration arguments. The

central focus of the early writings is on refutation arguments,²¹ and they will be our focus here. Aristotle calls refutation arguments “dialectical” arguments. So I will call the theory of such arguments *dialectical*. The word “dialectical” has a tangled usage in Greek. In Aristotle’s case, it is in all its senses a technical term Woods & Hansen (2004). For our purposes here it suffices to give it the sense that it currently carries in present-day argumentation theory. Accordingly, an argument in the broad sense is a dialectical argument when it is a contest between dialogue-partners over some disputed proposition in which each participant has the objective of prevailing against the other.

Arguments in the narrow sense stand starkly apart. They are not social events. They are not events of any kind. They are finite sequences of linguistic objects which Aristotle calls propositions. When they meet certain conditions, they are *sylogisms*. The terminal member of a syllogism is its conclusion, and the remaining members its premisses. It is generally agreed that Aristotle understands a syllogism to have exactly two premisses.

A syllogism is a valid deduction satisfying some further requirements: (1) Its conclusion may not repeat a premiss or any statement immediately implying it. That is, the argument must be non-circular. (2) There may be no redundant premisses. That is, all premisses must be load-bearing. (3) The premisses must be internally and mutually consistent. That is, inconsistent premisses yield no syllogistic conclusions. (4) There may not be multiple conclusions.²²

Aristotle is the originator of the syllogism. Its originality is something that he stresses and is evidently proud of.

When it comes to this subject [the syllogism] it is not the case that part had been worked out in advance and part not; instead *nothing* [before it] *existed at all*. (*Soph. Ref.* 183b, 34-36; emphasis added)

Syllogisms were to play a breakthrough role in the management of a vexing problem with arguments in the broad sense. The problem is that these arguments instantiate the appearance-reality distinction. Good-looking arguments are often bad, and bad-looking arguments are sometimes good. The problem is that up to now there has been no principled and suitably general way to regulate this distinction, hence no reasonable prospect of constructing a sound general theory of argu-

ments in the broad sense. But with the new idea of syllogisms now in hand, Aristotle thinks that prospects improve significantly. He will give to syllogisms a core role in a general theory of two-person, psychologically real, real-time arguments. I will illustrate how this works for dialectical arguments.

A dialectical argument is a dialogue about a disputed proposition called a thesis (T). It is a contest between a supporter of T (the proponent) and a rejector of it (the opponent). Once T has been advanced by its proponent P, the lead-role passes to the opponent O. At each stage of the dialogue, it is O’s task to put to P a single question which admits of a complete answer: Yes or No. The propositional content of that response then becomes available to O for future use, to be described immediately below. So it is necessary for the parties to keep track of these propositions.

O has a second task to perform. He must attempt to produce a syllogism whose conclusion is the contradictory, not-T, of P’s initial thesis.²³ A restriction on this syllogism is that all its premisses are to be drawn from the inventory of those propositions conceded by P in responding to O’s Yes-No questions. When these conditions are met, the syllogism that O constructs is a *refutation* of the disputed thesis.

A dialectical argument is an argument in the broad sense. The opponent’s role in it is to produce an argument in the narrow sense - that is, a syllogism - in fulfillment of these further conditions. A refutation is the syllogism that wins a dialectical argument for O. When this happens, the loser P stands convicted “out of his own mouth”. It is a clever requirement. It ensures that no premiss of a refutation ever begs the question against its proponent.

It is easy to see that the refutation of a thesis (A, B, not-T) does not establish its falsity. The most that the refutation discloses is the falsity of at least one member of the set {A, B, T}. But there is nothing in the structure of these proceedings that allows us to pick out the falsity(ies) within. What the refutation shows is that T is not consistently assertible by P in the context of the very dialogue, D, in which P asserted it. It shows that P has made an inconsistent defence of T in D. In so saying, we come to an important feature of Aristotle’s dialectical logic. A dialectical success is not, just so, an alethic success.

In various places Aristotle characterizes refutations as *ad hominem*

proofs. In so saying, he achieves an insight into *ad hominem* argumentation that has eluded virtually every modern writer with an interest in such matters.²⁴ Aristotle sees that an *ad hominem* argument can be a legitimate defeater of a thesis well short of showing it to be false, hence without its being a proof “in the full sense.” One defeats the thesis by inducing its supporter to spoil his own defence of it. In the Socratic literature, refutations are considered *negative elenchi* (Vlastos 1982).²⁵ Given the refutation $\langle A, B, \text{not-}T \rangle$, Aristotle thinks that the inconsistency of $\{A, B, T\}$ constitutes a refutation of T , not of A or B . The inconsistency “pins the blame” on T , notwithstanding that it doesn’t convict T of falsity. What justifies his singling out T in this way? Since Aristotle does not address this question directly, he provides no straightforward answer to it. That he thinks a refutation is a negative *elenchus* is amply present in his insistence that a refutation of T is not a proof against it “in the full sense”, but rather a proof against the person who holds it. Even so, the early writings provide the wherewithal for a dialectical answer to our question.

One of the requirements of dialectical argument is that answers may not be retracted. Call this the no-retraction rule. There is a reason for this. If answers could be retracted strategically, they could be pulled out of contention at any point in the argument at which their presence portends damage to the answerer’s thesis. Strategic withdrawal is precluded by three of the dialectical rules of procedure. One is that questions must be answered honestly. Another is that a proponent’s only move is to answer a Yes-No question affirmatively or negatively. He cannot retract T unless there is an askable Yes-No question, a negative answer to which is directly inconsistent with T . But, thirdly, if this happens, it violates the rule that answers may not contradict one another.

It bears on this that the only concession that is eligible for P to make is T itself. This is because P ’s endorsement of T occurs independently of any question his opponent asks of him in an argument that is *occasioned* by that prior espousal. By process of elimination, when P lands himself in a pickle generated by the inconsistency of $\{A, B, T\}$, T is the only proposition eligible for retraction. So the blame is pinned on T by the dialectical rules of refutation arguments. We may say, then, that when $\langle A, B, \text{not-}T \rangle$ is a syllogism that the refuted proposition T is *dialectically individuated*.

It is well to emphasize that a refutation is a syllogism. It owes nothing of its identity as a syllogism to dialectical considerations. It does however owe its identity as a *refutation* to dialectical considerations. As we see, a refutation’s dialectical features suffice for the individuation of the thesis that it refutes. It would be interesting to see whether this identification might be also achieved by way of a refutation’s *sylogistic* features. Let us see.

When $\langle A, B, \text{not-}T \rangle$ is a refutation of P ’s thesis T , then $\{A, B, T\}$ is an inconsistent set, and $\{A, B\}$, $\{A, T\}$ and B, T are its maximal consistent subsets. This we know because, by the structure of the syllogism, premisses must be internally and pairwise consistent, and no premiss can give not- T in one fell swoop. So $\{A, T\}$ and $\{B, T\}$ must also be consistent. That they are maximal is shown by the inconsistency of $\{A, B, T\}$, which is what the addition to each of these subsets of the proposition that is missing produces.

Let us say that a maximal consistent subset of $\{A, B, T\}$ is *excluded* by a refutation $\langle A, B, \text{not-}T \rangle$ just in case it doesn’t syllogistically imply the refutation’s conclusion. So defined, $\{A, B\}$ is not excluded by $\langle A, B, \text{not-}T \rangle$, but each of $\{A, T\}$ and $\{B, T\}$ is. Indeed T is *the* proposition refuted by $\langle A, B, \text{not-}T \rangle$ because it is the sole member of the intersection of all maximal subsets of $\{A, B, T\}$ excluded by the refutation. Thus it would appear that the positive refutation thesis can be upheld on syllogistic grounds. We might even say that it is *sylogistically individuated*.

It is important to see that in these writings Aristotle is drawing upon two different theories of argument. One is a theory of narrow arguments. The other is a theory of broad arguments. The narrow theory is the logic of syllogisms. It is a logic entirely free of dialectical characterization. Let us repeat the point that calling a syllogism a refutation is giving it a dialectical characterization. But it is not a characterization that the narrow logic, the logic of syllogisms, can bestow. It is not in the vocabulary of the logic of syllogisms to call a syllogism a refutation even when it is one. We need the vocabulary of a theory of broad argument to call a syllogism that. The broad theory, typified by the theory of refutation, has an even larger theoretical range, extending to any form of dialogue whose logic embeds the syllogistic as a partner logic. But for our purposes dialectic can stand in for them all. It remains only to add that Aristotle thinks that the logic of syllogisms

and the dialectic involve quite different conceptions of argument. The term “logic” didn’t arise until late in the second century A.D.²⁶ Aristotle instead speaks of “analytics”, which he reserves for the narrow logic. This is not a matter of baptismal haphazardness. In the distinction between analytic and dialectic Aristotle intends a difference in kind.

The distinction between dialectic and analytic (or logic) is clear in Aristotle.²⁷ In later writings, including those of the present day, the distinction is often blurred. But it is not non-existent. Provided we attend to the relevant differences perhaps there is no great harm in distinguishing between Aristotle’s *dialectical logic* and his *sylogistic logic*. But we should not lose sight of the point that these are disjoint conceptions of logic.

Aristotle’s breakthrough insight was the discovery that dialectic cannot succeed without a properly wrought partnership with logic. He thought that the partnership couldn’t succeed if the deep differences in kind between arguments in the broad sense and arguments in the narrow sense weren’t duly heeded. Neither could it succeed in the absence of a clear-eyed appreciation of the deep differences in kind between the “logic” of arguments in the broad sense and the logic of arguments in the narrow sense. Still, for all their differences, the partnership is an intimate one. The logic of syllogism will be the indispensable theoretical core - indeed a proper part - of a successful dialectic of arguments in the broad sense.

Even so, the distinction between dialectical and syllogistic logic engenders a complication. Marion’s challenge is now two challenges. One is to search the ancient records to see whether Aristotle’s dialectical logic is foundational for modern game theoretic logic. The other is to determine whether these roots are discernible in the syllogistic. The same is true of Marion’s optimistic suggestion that the search will prove successful. It is now an ambiguous optimism. In what follows, it behoves us to keep these complications in mind.

7. FALLACIES

Let me return to the point that, with his contemporaries and predecessors, Aristotle was worried about the appearance-reality distinction. It is a given that sometimes things aren’t in reality as they appear to be.

The general problem posed by this is how to regulate the distinction between *being* and *appearing* to be in a suitably general and principled way. As applied to arguments, there are good-looking arguments that aren’t good in reality, and bad-looking arguments that are in fact good. Aristotle thought that a suitably powerful general theory of argument would be one that dealt effectively with the distinction between actually good and merely good-looking arguments.

As we saw, Aristotle contrived the logic of syllogisms to play a central and load-bearing role in this general theory. As was evident from the goings on in the *agora* and the councils of government, disputations often descend into wrangles, and even on those occasions when they appear to have been settled, the appearance of settlement isn’t always the real thing. In the matter of refutation, Aristotle thought that the appearance-reality distinction could be regulated by the presence or absence in an apparent refutation of a properly constructed syllogism.

Syllogisms were purpose-built to make the appearance-reality distinction for arguments a manageable one. A refutation would be “sophistical” if it lacked the *bona fides* of a syllogism. As its title suggests, in *On Sophistical Refutations* all is not well with the logic of refutation.²⁸ An argument can appear to be a syllogism without being one in fact. Aristotle calls such arguments *paralogismoi* or “fallacies”. Fallacies are arguments that evade the discipline of the appearance-reality distinction for arguments. The irony of this cannot have been lost on Aristotle. Syllogisms were invoked to dispel appearance-reality confusions but, as we now see, this is the very confusion to which they themselves sometimes fall.

This matters for dialectical logic. Aristotle sees a would-be refutation as sophistical when it is fallacious; that is, when it has the look but not the reality of a syllogism.²⁹ Much of *On Sophistical Refutations* is a discussion of the various ways in which a non-syllogism might take on the look of a syllogism. Aristotle lists thirteen of the ways in which this confusion can arise: They are: equivocation, amphiboly, combination of words, division of words, accent, forms of expression, *secundum quid*, consequent, non-cause as cause, begging the question and many questions. There arises now a challenge of considerable difficulty. It is to establish an essential connection between two kinds of argumentational misperformance. On the one hand, it is necessary to expose the

conditions under which the thirteen pathologies on Aristotle's list are instantiated. It is also necessary to show that, when a would-be refutation instantiates such a pathology, this fact is necessary and sufficient for conferral of the appearance of syllogisticity on arguments that aren't in fact syllogisms.

It is a heavy burden, with respect to which Aristotle himself makes next to no progress. This alone is reason to wonder whether the essential connection between the thirteen pathologies and the fallacy of mistaking a non-syllogism for a syllogism can actually be established. Consider a case. If an opponent seeks to score against his adversary by extracting from him an admission of dog-beating, it is unavailing to ask, "Are you still a dog-beater?" At least, it is unavailing to ask it under the requirement that the answer must be either Yes or No. If it is Yes, the point is scored. But if the answer is No, the position its answerer has conceded let's simply assume the answerer possesses a dog is that either he didn't beat his dog in the past, or did but doesn't now, or didn't and still doesn't. It is not an admission that excludes the very point that the questioner wishes to score. It generates an answer which leaves it unproved that the answerer is indeed a dog-beater. So while it denies the answerer a defence against the charge, the answer it receives leaves the questioner's intended point unscored. The question was both unfair to the answerer and unavailing for the questioner. It was the wrong question to ask.³⁰

It also happens that it was a syllogistically inadmissible question. The statement conveyed by its No-answer fails to be a categorical proposition. It is unavailable for premissory work in any syllogism the questioner might have it in mind to construct. However, the point to emphasize is that the dialectical ineffectuality of the question owes nothing to its ineffectuality in generating a syllogically allowable premiss for the answerer's subsequent use. What makes the question dialectically ineffectual does not depend on whether its asker has any syllogistic designs upon it. That being so, a question is not a damage free maneuver in the many questions sophism simply because it does not lend itself to inclusion in a would-be syllogism premissed in part by the question's No answer.

This is not to say that Aristotle can't reinstate the connection he seeks by *fiat*. This is, in fact, precisely what he does in the early writ-

ings. Aristotle stipulates that a refutation is a syllogism meeting certain conditions. Accordingly, we have it at once that no argument that fails to be a syllogism can succeed as a refutation. Aristotle further stipulates that a fallacy is an argument that has the false appearance of a syllogism. A third stipulation is that a would-be refutation is sophistical when it is such as to occasion the false appearance of a syllogism in an argument that fails to be one. In our present example, the fallacy sophistically achieved by many questions consists entirely in masking or rendering unapparent the fact that it *does not* achieve a syllogistic refutation by means of the premissory employment of a No-answer.

This is getting to be an odd tangle of stipulations. Intuitively, the impropriety of many questions inheres in its unfairness to the answerer and its dialectical impotence for the asker. Intuitively, the question is bad dialectics even before it receives an answer and notwithstanding the questioner's further designs upon it once made. If the same were true of the twelve other pathologies on Aristotle's list, we would have it that Aristotle's *list* of the fallacies fails to instantiate his own *concept* of them.³¹ But Aristotle wants to give his invention a central and load-bearing role in dialectic. He wants dialectic to pivot on logic. And so it does in these writings; but it does so by Aristotle's contrivance.

The fallacies project arose with the very birth of logic itself. It was not an easy birth. Some would go further: it was a still-birth. The history of logic attests to this in a number of ways, two of which I'll mention here. One is the recurring tendency among commentators on Aristotle's fallacies programme to interpret it as a dialectical project, in a sense of "dialectical" that sets it apart from logic.³² The other, not unrelated, historical indication is reflected in two further facts about the programme. Aristotle himself did little to bring it to theoretical maturity,³³ and so too the programme's successors, a point lamented in Charles Hamblin's *Fallacies* (1970).³⁴ Indeed, as logic has evolved not excluding the expanding markets for dialogic logic the fallacies project has largely dropped from view. (When did a paper on the fallacies last appear in the *Journal of Symbolic Logic*?)³⁵

What this suggests is that the mischief actually done by Aristotle's thirteen pathologies has nothing inherently to do with logic in the narrow sense. Even so, Aristotle fashions an attachment by fiat. Aristotle didn't set out to dialecticize logic. He invented logic to *logicize dialectic*.

tic. If this is right, it is a primacy to respect before reaching a judgment about Aristotle's game theoretic *bona fides*.

8. CONSEQUENCE HAVING AND DRAWING

Like all logics, the central focus of syllogistic logic is the consequence relation, called by Aristotle necessitation. Strangely enough, necessitation is primitive in Aristotle's logic. Aristotle's focus is on what we might call *sylogistic consequence*. In the *Prior Analytics*, Aristotle is quite explicit about the distinction. A valid argument is *anagkaion*, and *sullogismos* is a special case of *anagkaion* (*Pr. An.* A 32, 47a). Syllogistic consequence is the necessitation relation under the constraints that define syllogisms. It is an open question as to how closely necessitation resembles modern notions of consequence, e.g. classical consequence. In fact, for the purposes of syllogistic logic, it can be any relation that meets the demands of truth-preservation. If we make the not implausible assumption that none of Aristotle's syllogistically defining properties need be required for necessitation, necessitation certainly could be classical. The point is that the syllogistic conditions are sharply constraining ones. Here is why.

Implicit in these writings is a distinction between consequence-having and consequence-drawing. In a great many systems of modern logic, it is taken as given that any consequence of any premisses accepted by a reasoner is a consequence he should draw. This is a ludicrous requirement for real-life reasoners. For one thing, there are infinitely too many of them, and, for another, vanishingly few of them would have any conceivable interest for or would confer any conceivable benefit on their drawers. This has generated a separatist movement in logic: A logic of consequence-having is one thing. A logic of consequence-drawing is another. Accordingly, if there are logics of such things, they will have to be different logics. (Separate, but equally logics, so to speak). A further option is to close the gap between having and drawing by normativizing the consequence-having rules for ideal reasoners. In which case, the ideal reasoner would close his beliefs under consequence.³⁶

That Aristotle has no definition of consequence suggests that his entire orientation in the early writings is on consequence-drawing, that is,

on the sort of consequence-drawing appropriate to dialectical contexts. This he achieves by first defining a restricted notion of consequence - syllogistic consequence - and then by closing the gap between having a syllogistic consequence and drawing it. This is entirely an outcome of how syllogisms are structured. Essential to this gap-closing is Aristotle's insistence that the propositions from which syllogisms are constructed be what would later be called categorical propositions, propositions in the classical A, E, I and O formats. This, too, is a harsh constraint, for which, beginning with the Stoics, Aristotle would be rebuked. But, it is no ad hoc contrivance. Aristotle thinks that every statement of Greek can be re-expressed without relevant loss in the language of categorical propositions.³⁷

The net effect on the question before us of the conditions that transform valid arguments into syllogisms is this: For any arbitrarily selected pair of propositions, hardly any has syllogistic consequences at all; and when they do, the consequences are never more than two. Aristotle requires that a syllogism contain exactly three terms, each of which occurs exactly twice (but not in the same proposition). This, together with the two-premiss limitation and one-conclusion rule, greatly inhibits syllogistic output. If C is the syllogistic conclusion of premisses A, B, it is their only conclusion unless a further proposition D is immediately implied by C by subsumption or immediately equivalent to it by conversion. Thus if C is "All Greeks are mortal", D could be "Some Greeks are mortal". If C is "Some felines are cats", D is "Some cats are felines." This tightened syllogistic structure serves to close the gap between consequence-having and consequence-drawing. Under the syllogistic constraints, it is wholly reasonable to require the acceptor of a syllogism's premisses to draw every *sylogistic* consequence of them. Accordingly, the idea that logic should serve two masters at once - having and drawing - receives its first accommodation at the subject's very beginning.

Logics of various stripes have also tried to close this gap. In modern terms, the gap between having and drawing is the gap between implication and inference. Think here of relevant and other forms of paraconsistent logic. Although, in comparison to classical logic, these have a more gap-closing character, even here the rules of inferences remain unexecutable by beings like us. They are rules for the ideal

reasoner. Aristotle closed the gap for *human* reasoners. No mainline modern logic has yet to do the same. Syllogistic is the first and most successful gap-closing logic in the venerable history of the subject.

In marrying the logic of syllogistic consequence and the logic of syllogistic inference, Aristotle is taking account of the cognitive finitude of the human reasoner. The motive for this is entirely straightforward. Logic in the narrow sense is a service industry. (After all, *organon* means “tool”.) It is the premiss-consequence engine that serves the further processes of logic in the broad sense. One of the dialectical rules obliges the proponent to accede to all the syllogistic consequences of any pair of propositions conveyed in answers to his opponent’s Yes-No questions. One of the narrow logic’s central contributions to the logic of arguments in the wide sense is that this dialectical obligation is performable by psychologically real agents in real time.

All this matters greatly for the logic of arguments in the broad sense. When one party is contending with another, and when success or failure depends on whether the one party can get the other to see that a given proposition has to be drawn in consequence of the other’s own concessions, it is useless that the proposition in question does in fact follow from them if the other is unable to draw it as such.

9. FOUNDATIONALITY

One of our questions is whether Aristotle’s logic is foundational for the logic of games. It is, as we saw, two questions, one for the dialectical logic, the other for syllogistic logic, which is dialectical logic’s theoretical core. Let me begin with the core logic. The non-circularity condition immunizes syllogistic reasoning against one source of question-begging; modern logics of consequence typically do not. The premiss-nonredundancy condition imposes on syllogistic consequence a relevance condition similar to but stronger than the Anderson-Belnap full-use sense of relevance [Anderson & Belnap \(1975\)](#). It also provides for the nonmonotonicity of syllogistic consequence, which is intolerant of extra premisses. That same constraint, together with the premiss-consistency requirement,³⁸ constitutes the syllogistic logic as a limiting case of a paraconsistent logic.³⁹ The ban on multiple conclusions also suggests a connection to intuitionism ([Shoemith & Smiley 1978](#), p.

4). Taken together, the logic of syllogisms is a relevant, paraconsistent, nonmonotonic logic, executable in dialectical and other dialogical contexts by beings like us in the actual here and now.

Except for the metaphor of being in Aristotle’s logical DNA, I haven’t had much to say about this property of foundationality. It is now time to say something more, beginning with nonmonotonicity. A consequence relation is nonmonotonic when it is open to erasure in the face of new premisses. Such relations are, let us say, “bustable”. Bustability is a necessary feature of the syllogisticity of a consequence relation and a necessary and sufficient condition of the nonmonotonicity of a consequence relation. Accordingly, we might propose that

Foundationality (First version): One system is in the logical DNA of another when there is some property necessary for a feature of the first logic’s consequence relation that is necessary and sufficient for a feature of the second logic’s consequence relation. The first logic, then, is foundational for the second.

It is easy enough to see that this connection obtains between the logic of syllogisms and most of the mainstream logics of paraconsistency. The failure of the *ex falso quodlibet* theorem is a necessary feature of the syllogisticity of consequence and a necessary and sufficient condition of the paraconsistency of consequence.⁴⁰ So the logic of syllogisms is foundational for paraconsistent logics. Similarly, full use of premisses is a necessary condition of syllogisticity and a necessary and sufficient condition of the full use relevance of Anderson’s and Belnap’s proofs from hypotheses. The question we must now ask is whether, in this same sense or anything convincingly like it, the logic of syllogisms is foundational for the modern logic of games.

Of course, the Foundationality Principle is only a suggestion, scarcely more than a passing idea. While it might plausibly enough capture one sense of theory-foundationality, there may be other foundationalities that exceed that principle’s reach. One thing we don’t want it to obscure is the possibility that when, in respect of some shared property or feature, one logic is foundational for another, this needn’t derive from a common *motivation*. The paraconsistency property is an example. Modern paraconsistentists will tolerate valid arguments with

inconsistent premisses. What they will not tolerate are consequence relations that take inconsistent premisses to the lengths of absolute inconsistency. Accordingly, paraconsistent logicians place requisite constraints on consequence. The effect of these constraints is to keep localized inconsistencies in their place, not eliminate them altogether. On the other hand, Aristotle would not tolerate inconsistent premisses in the first place. Inconsistently premisses valid arguments have circular counterposes. Since circular arguments aren't syllogisms and the contraposition relation is syllogisticity-preserving, neither is an inconsistently premisses argument a syllogism. Advocates of paraconsistent logics cannot achieve their ends with classical consequence. But Aristotle conceivably could. Aristotle deals with his problem not by trifling with the necessitation relation. He achieves it by restricting premiss-eligibility. Both logics are paraconsistent in the sense that neither tolerates *ex falso*. But what they *aren't* are logics whose respective paraconsistency has a shared motivation. We can say this more directly. Whatever the similarities, Aristotle certainly didn't set out to be a da Costa or a Routley.

The same can be said of some of the other shared features. Consider again the case of nonmonotonicity. Modern nonmonotonic logicians widely assume that a consequence relation's being nonmonotonic denies it the property of being truth-preserving. Nonmonotonicity has a quite different impact on syllogistic consequence. It shows that a valid argument that happens to be a syllogism is not, upon addition of new premisses, syllogisticity-preserving. But it does not show, nor is it likely to be true, that that same argument in those same circumstances is not truth-preserving. This is a significant difference, needless to say, and it may lead some readers to think that the nonmonotonicities of Aristotle and the modern non-deductive logician have quite different motivations. In other words, Aristotle didn't set out to be a Reiter or a McCarthy or a McDermott and Doyle.

Perhaps it is not quite this way with relevance, whether expressed as Aristotle's premissory nonredundancy or as Anderson and Belnap's "full-use of hypotheses". They both arise from a shared dislike of lazy premisses, of propositions that are surplus to need. Yet the same could not be said of Anderson and Belnap's further notion of relevance, sometimes called the "content-containment" sense. In this instance, their

motivation was to avoid irrelevance. Aristotle's was to avoid redundancy, never mind that a redundant premiss might well be relevant in something very like the content-containment sense. Anderson and Belnap worried about the damage done to the classical consequence by the absence of concept-containment. Aristotle need have no such worry about classical consequence. His is a worry about syllogisticity. And his interest in syllogisticity is born of a desire to close the gap between consequence-having and consequence-drawing.

On its first or weak version, Foundationality favours a positive finding on the foundational significance of the syllogistic for the properties under review. But if we elected to strengthen the principle by adding a commonality of motivation requirement, the foundationality claim would be considerably weakened, indeed pretty much wrecked.

This clearly matters for the game theoretic foundationality, or want of it, in Aristotle's work, and adds a complication. When we raise this question, we raise it for two logics, not one; and we raise it for two senses of foundationality, weak and strong. Perhaps we can simply concede that the strong version will be the harder sell. But neither should we think that the weak version is free-on-board.

10. GAME THEORETICALLY FOUNDATIONAL?

Let us start with the syllogistic. A game theoretic logic is a logic governed by logical and structural rules. The logical rules interpret the logic's connectives and quantifiers and such other logical expressions as it might have. Structural rules are of organizational and strategic import. They fix conditions for the start of play, for transitions from one stage to another, and they specify what counts as a winning strategy. This makes possible the definition of properties such as validity and proof. To qualify as game theoretic, a logic must be governed by rules of both types. If a logic lacks logical rules in the sense intended here, the question of its game theoreticity is settled in the negative. So we must ask, "Is the logic of syllogisms governed by logical rules of the kind specified in section 1?"

It is hardly plausible that Aristotle defined his logical particles at all, that is constructed a semantics for them. Still, it would be interesting to know whether, had he done so, he would have been guided

by game theoretic, as opposed to (say) truth conditional instincts. The logical terms of the syllogistic vocabulary are “all”, “some”, “not” and “is”.⁴¹ The nonlogical expressions are terms, representable by schematic letters. “All” and “some” prefixed to terms give a kind of quantification of the term. “Not” is term complementation, yielding further terms. It can also prefix the quantifier “all”. Terms admit of occurrence in both subject and predicate place. Singular terms are banned: “Socrates” is construed as “All that is Socrates” or “Every Socrates”.⁴² Quantifiers occur only as prefixes of subject terms. “Is”, the copula, is flanked by quantifications of terms on the left and terms on the right, the results of this are propositions. Categorical propositions are of four kinds “All S are P” (A), “All S are not-P” (E), “Some S are P” (I), and “Some S are not-P” (O). Propositions are linked by relations of immediate (not syllogistic) implication: A immediately implies I; E immediately implies O. In addition, A and O are one another’s contradictories. A and E are one another’s contraries. I and O are one another’s subcontraries. If two propositions are contradictories one is true and the other false. They are contraries when they can’t both be true but could both be false. They are subcontraries when they can’t be false together and yet could be true together. Since they hold between single propositions, contradictoriness, contrariety and subcontrariety are not syllogistic relations.

This essentially is the Aristotelian story of the logical expressions. It is set out compactly and schematically in the Square of Opposition, which arises not in the *Topics* and *On Sophistical Refutations*, but in the *Prior Analytics*. In particular, the Square defines “is the contradictory of” in such a way that instances are recognizable in the syntax of its relata. When, as in dialectical arguments, the opponent’s role is to find a syllogism whose conclusion is the contradictory of the thesis under attack, all needed information about contradictoriness is at hand. But it would be a strain to say that this information has been produced game theoretically. The point generalizes. Whatever the information Aristotle provides about term complementation, term quantification, the copula, and the logical relations of immediate or one-step necessitation (which Aristotle also calls subsumption), contradictoriness, contrariety and subcontrariety, there is in the DNA of these works no discernible presence of a game theoretic sensibility, even in the weak sense of founda-

tionality.

If this is right, the logic of the syllogism, whether in its comparatively fledgling development in *Topics* and *On Sophistical Refutations* or in its developed form in *Prior* and *Posterior Analytics*, fails to meet the test of game theoretic foundational significance. It is hardly surprising that this would be so. It is true that syllogisms were *designed* for load-bearing work in dialectical arguments, but they were not themselves *defined* dialectically. For the most part, the dialectical rules are constraints. They offer scant positive guidance about, for example, how to ask *telling* questions, that is, questions that yield the premisses for a refutational kill. Syllogistic conditions, on the other hand, make the property of syllogistic readily recognizable, if not actually decidable.⁴³ If syllogisms are not dialogical entities, less so are they dialectical entities. If they are not dialectical entities they can hardly qualify as game theoretic entities. This suffices to deprive the logic of the syllogism of any trace of a language of winning and losing. So the game theoretical hypothesis loses traction here.

Some will say that this is all well and good for syllogistic logic, but isn’t dialectical logic another matter entirely? Dialectical logic is a logic in the broad sense. Like all logics in the broad sense, it is a logic of *arguing*. In its dialectical variation, it is a logic of contestation, a logic of interpersonal confrontation and rivalry. Whatever its details, how could a dialectical logic not resonate with game theoretic purport?

There is procedural guidance in Aristotle’s dialectic, much of it contained in *Topics*, book 8. There is, as we saw, one proponent and one opponent. The proponent must defend a thesis by answering the opponent’s question, posed one at a time. The questions must be clear and straightforward, and should include what in law are called leading questions. Questions not meeting these conditions can be refused. Questions must be fully answerable by an answer of Yes or No. The content of such an answer must be expressible as a single categorical proposition. Proponents must believe their answers to be true. Opponents are free to use the propositions conceded by their respondents, but they need not believe them to be true. Answers may not be postponed. Answers may not be withdrawn. If a proponent doesn’t know the answer to a question, the opponent and he must enter into an instruction argument that removes this ignorance. Since the opponent’s

task is to attack the proponent's thesis by producing a syllogism whose conclusion is its contradictory, he should contrive his questions in ways that facilitate that outcome. The argument ends when such a syllogism is produced, or with the joint recognition of the opponent's failure to produce it. The first ending is a win for the opponent. The second is a win for the proponent, akin to the Scotch verdict of "not proven".

The language of Aristotle's dialectic is replete with the idioms of games, of winning and losing, of procedural entitlements and prohibitions, including those that help regulate the generation and selection of premisses and conclusions. Aristotle's dialectic is redolent with strategic purport, and it is offered to real-life Athenians for the (somewhat idealized) amelioration of real-life disputational turbulence. But the question is whether, as they occur there, the idioms of game-playing carry the sense, or some fair adumbration of it, that they possess in the logics that descend from von Neumann and Morgenstern.

Let me repeat an earlier disclaimer. I said that one of the questions I wasn't interested in asking is whether a game theoretic interpretation couldn't be imposed on Aristotle's dialectic. Consider a case. There are logicians aplenty - John Venn being but one of them who interpret the logic of syllogisms as a logic of classes. There is perhaps no harm in it. But it is not very plausible to think that this was Aristotle's interpretation. Less plausible still would be any suggestion that Aristotle's postulated classes carry the sense of Cantor's sets. More briefly: Nobody in his right mind would think that a logic of syllogisms is foundational for modern mathematics, that Cantorian set theoreticity was in Aristotle's logical DNA. The same, I think, must turn out to be true of Aristotle's dialectic in relation to the modern games of logic. This, if right, fulfills Marion's prompting. But the scholarship thus occasioned would provide no encouragement of Lorenzen's conjecture and no answer to Hodges' question in Aristotle's writings.

11. SEMANTIC INHERITANCE

There are two foundationality questions to judge, weak and strong, and two logics with respect to make these judgements, syllogistic and dialectical. The gist of my remarks so far is that not even the weak foundationality claim for game theoreticity gets much traction, not even in

the dialectical logic. I take it as given that some readers will have found this negative case to be unconvincing. Perhaps this is so. Perhaps the negative thesis is just wrong. If the negative thesis is wrong, there must be a case against it, at least in principle. I find myself wondering what such a case might look like. Perhaps it might look like this.

After some initial dissensus about the logical character of the syllogistic, modern opinion now favours a proof theoretic natural deduction interpretation according to which Aristotle's logical particles are defined by proof rules. Consider, for example, the \forall as we would now call it-universal quantifier. In *Prior Analytics* A, 2, 24^k 28 we have it thus:

And we say that one term is predicated of all of another, whenever nothing can be found of which the other cannot be asserted.

Modern commentators, e.g. von Plato (2013), call this the "no-counterexample" interpretation. The no-counterexample reading is also discernible in passages of the *Topics*. At Θ , 2 157^a 34 Aristotle remarks:

If one has made an induction on the strength of several cases and yet the answerer refuses to grant the universal proposition, then it is fair to demand his objection.

This continues at Θ , 8, 160^b 3:

... against the universal one should try to bring some objection: for to bring the argument to a standstill without an objection, real or apparent, shows ill-temper
... If, moreover, he cannot even attempt a counter-proof that is not true, far more likely is he to be thought to be ill-tempered.

In other words, declining to give a counterexample to a disputed universal proposition is not quite cricket, not playing the game.

For logicians of game theoretic leanings, there are things to like in these passages.⁴⁴ For do they not support a game theoretic construal of the no-counterexample interpretation of universality? My answer is "Why would they?". True, the passages from *Topics* are expressly

concerned with how an opponent's universal claim might properly be countered. They advise the attacker on how to proceed against an opponent's claim when it possesses that logical form. We can say this another way. *Given* the meaning of "All A are B", the right way to proceed against it is to try to find a counterexample. But it does not follow, nor is it at all obviously true, that the proposed rules for rebutting a universal proposition (as well as the rules for sustaining it) give the meaning of "all". The rules call for the production of a counterexample. They do so precisely because the meaning of "all" requires a counterexample for the rebuttal of "All A are B". The meaning precedes the procedural rule and, in so doing, gives it a coherent motivation. Game theoretic logicians *reverse* this dependency. The prior fact, they say, is that this is the right way of rebutting a universal claim, and the meaning that "all" acquires is inherited from this and like rules. So we have here a clash of two *semantic inheritance* claims.

Do we have a ready means of solving the semantic inheritance problem? If so, let us solve it and let the chips fall where they may. One way of achieving this resolution, and of releasing those chips, would be to show that a dialecticized verificationism is the right semantics for our language.

Suppose that the dialecticized verificationism briefly sketched in section 3 were indeed the correct theory of meaning for "All A are B". Then we could say that the meaning of "all" is indeed constituted by our dialectical practices. This would give us grounds for two different claims. One is that the refutation advice offered in the early writings *honours* the meanings of the sentences involved. The other is that those very practices *constitute* their meaning. This is problematic. Consider any treatment of logic whose provisions for "all" honours the meanings that it actually possesses. By this I mean that nothing in the theory's quantification rules is vitiated by what "all" actually means. The quantification rules "do no logical violence" to the meaning of "all". Consider in particular the model theoretic approach of Tarski. No one seriously believes that Tarski's provisions for the logical particles does violence to what those expressions actually mean. But game theoretic logicians think that Tarski's theory of meaning is wrong - or anyhow subpar. If it followed from the fact that Tarski's logic does no violence to the meanings of these expressions that Tarski's semantics is a game

theoretic one, then Tarski would be a game theoretic logician *malgré lui*.

This brings us to the nub of the matter. Tarski wanted an accurate and theoretically deep account of consequence and logical truth. To achieve this he would specify a language with respect to which these would be definable properties. Tarski's characterization of them would depend on a prior characterization of the language's logical particles, \sim , \forall , and so on. Tarski's project was, in effect, to say what "is a consequence of" and "is a logical truth" mean. This, among other things, would involve his giving a meaning to " \sim ", " \forall " and the like. It is commonly supposed that he couldn't get the meanings of "consequence" and "logical truth" right unless he also got the meanings of " \sim " and " \forall " right, unless, that is to say, his " \sim " actually means "not" and his " \forall " actually means "all". On the assumptions currently in play, Tarski did not get the meanings of " \sim " and " \forall " right. How, then, could we suppose that he got the meaning of "consequence" right?

There is, as we now see, a critical difference between a logic that does no violence to the meaning of " \forall " and a logic that has the right theory of meaning for " \forall ". A logic that does no violence to the meaning of " \forall " is one whose transformation rules for " \forall " place -sentences in the correct deductive relationships. For this to happen, it is not necessary that the logic assign to " \forall " the meaning it actually has. The reason for this is that there is sufficient similarity between the game-theoretic "all" and the truth conditional "all" to slot "all"-sentences into the right consequence-contexts.⁴⁵

Perhaps a case can be made for the idea that the meaning that "all" has in the narrow logic is given game-theoretically by the dialectical rules of the broad logic in contexts such as *Top*. Θ , 2, 157^a 34, 8, 160^b 3. If this were so, it would matter quite a lot that the narrow logic is an essential constituent of the broad logic. This would help us see that even if there is nothing in the narrow logic's treatment of "all" that sustains the suggestion of a game theoretic orientation, it could still be true that in the narrow logic "all" occurs with the meaning given it in the dialectical logic, and that the dialogical logic is, nearly enough, a logic of games in the modern sense.

Yes, of course. But this is getting to be quite a gathering of "might"s. Dialectical verificationism might be true. If so it might be that the pro-

cedural rules of the *Topics* serve to fix the meaning of “all”. This would give us two cases to consider. One is that Aristotle himself believes that dialectical verificationism is true and that his rules in the *Topics* are meaning-constitutive. The other is that although dialectical verificationism is true, this is a truth of which Aristotle was entirely innocent. In which case, the rules of the *Topics* do indeed fix the meaning of “all” notwithstanding that nothing of the sort was in Aristotle’s mind. I have two things to say about this. The first is that there is no credible evidence either in Aristotle or the commentators that the founder of logic was any kind of semantic verificationist. So case one is dismissible without further pleadings. The second is that if in the *Topics* Aristotle is a meaning-fixer *malgré lui*, there is nothing to the strong idea that Aristotle intended the *Topics* to be meaning-fixing, as opposed to strategically instructive. Accordingly, the idea that Aristotle’s logic is game theoretically foundational loses a good deal of its steam. For we would have it then that Aristotle’s logic is a game theoretic semantics because the game theoretic approach is the right theory for “all” (etc.) and Aristotle’s logic does not - any more than Tarski’s does - logically violate the meanings that “all” (etc.) actually have. If this made Aristotle’s logic game theoretically foundational, how could it not do the same for Tarski’s logic?

Notes

¹ See also Nash (1950a; 1950b; 1951).

² Kuhn (1997) contains the classical foundational papers. For good general coverage, see Dixit et al. (1999). For mathematics, Fudenberg and Tirole (1991) is excellent. Gintis (2000) also covers mathematics but extends treatment to evolutionary game theory. La-Casse and Ross (1994) connects game theory to micro and macroeconomics. Schelling (1960; 1984; 2006) supply extensive coverage of social and political matters. Baird et al. (1998) applies game theory to jurisprudence, and Poundstone (1992) converts it to the strategic considerations that drove Cold War policy. Behavioural and experimental links are explored by Kagel and Roth (1995). Evolutionary game theory arises with Maynard Smith (1982), and its mathematical foundations are examined by (1995). Connections to neuroeconomics are treated by Montague and Berns (2002), and the links between game theory and behavioural economics and neuroeconomics are examined by Ross (2005) with a cognitive science orientation. Game theoretic versions of Rawlsianism are achieved by Binmore (2005). *Skyrms* (1996) gives a game theoretic treatment of social contract theory.

³ An important point of origin is Kuno Lorenz’s PhD dissertation (1961). Gale & Stewart (1953) connects the mathematics of game theory to set theory, and Henkin (1961)

to logic. See further Hintikka (1968, 1973, 1996), Lorenzen & Lorenz (1978), Barth & Krabbe (1982), Hodges (1985), Otto (1997), Rahman & Rückert (2001), Pauly & Parikh (2003), Rahman & Tulenheimo (2009), Peters & Westerståhl (2006), Rahwan & Simari (2009), van Benthem (2011), van Benthem et al. (2011), d’Avila Garcez et al. (to appear).

⁴ Hodges writes “ \exists ” (for “Eloise”), where others write “the proponent”. Similarly, where Hodges writes “ \forall ” (for “Abelard”), others write “opponent”.

⁵ See Lorenzen (1960), p. 187.

⁶ “For a large class of cases - though not all - in which we employ the word ‘meaning’ it can be defined thus: the meaning of a word is its use in the language.” (Wittgenstein 1953, p. 43).

⁷ As with “game” we cannot find “what is common to all these activities [= language games] and what makes them into language or parts of language.” (Wittgenstein 1953, p. 66).

⁸ Waismann (1967/1979) reports conversations in December 1929 and January 1930 in which Wittgenstein avers that “the meaning of a proposition is its method of verification”.

⁹ Or the tendentiously purported distinction between meaning and “cognitive” meaning.

¹⁰ I own to a slight reservation about over-extending the notion of game. The bloody victory at Vimy Ridge was a battle, sure enough. But the suggestion that it was also a game would have been a perilous one to make to the poor grunts who fought in it. Similarly, ask a soldier whether he thinks that war-games are a game, and one risks an unpleasant denial.

¹¹ For example, prisoners’ dilemma accounts of justice have an arguable presence in the presocratic record, in the Socratic dialogues, and in later writers such as Lucretius. See Denyer (1983).

¹² See for example Łukasiewicz (1957), Patzig (1968), Corcoran (1974), Lear (1980), Thom (1981), and Boger (2004).

¹³ See for example McCall (1963), McKirahan (1992) and Patterson (1995).

¹⁴ So-called by Bochenski (1970). “Immature” is ambiguous between “developmentally early” and “callow”. I am not sure which was Bochenski’s intent, but mine is the former, not the latter. The logics here considered are “early”, in contrast to the “later” developments of *Prior* and *Posterior Analytics*.

¹⁵ See Woods (2001) and Woods & Hansen (2004). But here, too, “earlier” and “later” have less a chronological significance than a developmental one. Some scholars have *Posterior Analytics* I as pre-dating both *Topics* and *On Sophistical Refutations*. Since this part of *Post. An.* is itself a considerable anticipation of formal developments in *Prior Analytics*, it is hard to see how the “early” logic could have been thought up in complete independence of the “later” logic. See the editors’ introduction to the Loeb edition of *On Sophistical Refutations*, and *On-Coming-to-Be*, and *On the Cosmos*.

¹⁶ Hamblin (1970) and Barth & Krabbe (1982).

¹⁷ Hintikka (1968, 1973, 1996, 1987, 1989, 1981).

¹⁸ Hodges (2004, §3), Marion (2009, p. 7), and Tennant (1979).

¹⁹ I might note that Marion himself has developed two answers to Hodges’ unvoiced question. One is worked up in an interesting adaptation of a Dummett-Brandom semantics (2009, pp. 19-23). The other is his contribution to this *Yearbook*. See also Marion &

Rückert (forthcoming).

²⁰Thus Quine: “Logic is an ancient subject, but since 1879 it’s been a great one.” (Quine 1950/1982, preface).

²¹Refutation arguments are not original to Aristotle. They are prefigured in the Socratic notion of *elenchus*. See Robinson (1953), Vlastos (1982) and Tuozzo (2011).

²²Aristotle’s definition, early and late, provides that: “A syllogism rests on certain propositions such that they involve necessarily the assertion of something other than what has been stated, through what has been stated.” (Soph. Ref. 165a 1-3) For an examination of whether structures so defined are able to bear my interpretation of them, the reader may wish to consult Woods (2001) or more briefly Woods & Irvine (2004). Some of my claims are open to dispute. For example, against the two-premiss condition, there are instances in which Aristotle himself cites a one-premiss argument as a syllogism. See Hitchcock (2000) for details. My view is that they are slips rather than counterexamples.

²³“Not-T” is a notational convenience of my own. It is not intended to reflect the logical form of T’s contradictory, but only to denote it whatever its form. So, for example, if T is the proposition “All A are B”, then not-T is “Some A are not B”.

²⁴A notable exception is Locke (1970).

²⁵While this is the standard view of scholars of the period, some see in Plato’s *Charmides* an account of *positive elenchi*. (Tuozzo 2011).

²⁶In a coinage of (Alexander of Aphrodisias 1881, 1883).

²⁷This is not to overlook the logic of immediate inference, more accurately described as the logic of single-premiss implication. Of course, no one-step implication is a syllogism. Syllogisms must have more than one premiss. Aristotle thought that valid single-premiss arguments were question-begging (*Top.* 162b 34). It is therefore more accurate to characterize Aristotle’s notion of logic as a family of theories of implication defined over arguments in the narrow sense, not excluding single-premiss arguments. For present purposes, however, the dialectic-syllogistic distinction will suffice.

²⁸Aristotle sets out the scope of *On Sophistical Refutations* in these words: “First we must grasp the number of aims entertained by those who *argue as competitors and rivals to the death*. These are five in number, refutation, fallacy, paradox, solecism, and fifthly to reduce the opponent in the discussion to babbling - i.e. to constrain him to repeat himself a number of times: *or to produce the appearance of each of these things without the reality*” (165b 12).

²⁹This requires a slight amendment. When considering sophistical refutations of the *ignoratio elenchi* sort, Aristotle allows that the argument at hand might well be a syllogism derived from dialectically allowable premisses, but whose conclusion fails to be the contradictory of the thesis of the refuter’s opponent. In which case, the *paralogismos* would be mistaking the non-contradictory of a proposition for its contradictory.

³⁰This is also part of the motivation of the requirement that syllogisms not have multiple conclusions.

³¹This idea, adjusted to modern fallacy theory, is developed in Woods (2013).

³²See Hamblin (1970), and again Hintikka (1987). For a contrary view see Woods & Hansen (1997), and Woods & Hansen (2004). Hintikka (1997) is a rejoinder to Woods & Hansen (1997).

³³Let me hastily mention a possible explanation of this. In the *Prior Analytics*, Aristotle produces an almost sound proof of the perfectability, a proof whose repair lies within

ready reach Corcoran (1972). A syllogism is perfect when its syllogisticity is obvious, when it wears its syllogisticity on its face. The perfectability proof provides that any syllogism whose syllogisticity is not immediately apparent can be shown to be a syllogism in a way that uses rules whose legitimacy is in turn immediately apparent. Therefore, the perfectability proof is a principled means for regulating the distinction between syllogisms and non-syllogisms. If this is right, the fallacies problem has a solution in the *Prior Analytics*. But it is a solution that bears no essential connection to the fallacies on Aristotle’s list.

³⁴survey of the history of fallacy theory in western logic can be found in Woods (2012).

³⁵At a certain level of abstraction and generality this historical observation is true. Closer to the ground, however, events take on a degree of complexity. In 1970, Hamblin would challenge the logic community to revive the fallacies programme. An early response was the so-called Woods-Walton Approach advanced in a series of some twenty-five papers published in the period 1972-1985. A distinctive feature of the Woods-Walton Approach was its attempt to adopt various nonclassical logics - notably intuitionistic logic but not excluding the formal logics of dialogue - to capture various properties peculiar to the various fallacies. Most of these papers are collected in Woods & Walton (1989/2007). Of its nineteen chapters, five first appeared in mainstream logic journals and four others in leading journals of technically oriented analytic philosophy. Another development of note, also in some measure a response to Hamblin, was the rise of informal logic, so-called. Although not universally instantiated, by far the dominant methodological emphasis of this movement is its eschewal of - indeed its hostility toward - formal methods in the theory of argument. With scant exceptions, informal logicians publish their work in niche journals such as *Informal Logic*. The motivation and development of this approach to argument is well described in Johnson (1996). This is not, I repeat, to overlook the robust attachment of formal developments in dialogue logic to the analysis of argument. But comparatively little of that work is devoted to the fallacies. The fallacies, if anything, are an afterthought.

³⁶See, for example, Harman (1970). Actually Harman is an extreme separatist. He reserves the name of logic for theories of consequence, and withholds it from theories of inference or consequence-drawing. Harman, like Quine, is a logical monist, and a strict conservative. He thinks that the only logic that deserves logic’s name is classical first order logic Harman (1972). Hintikka, on the other hand, is a moderate separatist. In his information-seeking interrogative logics, a distinction between *definitory* and *strategic* rules is fundamental. Roughly speaking, the definitory rules serve the needs of consequence-having, and the strategic uses govern the business of consequence-drawing. Moderate separatism is also proposed by Woods & Walton (1989/2007). Still, gap-closing normative idealization remains much the preferred option. For reservations, see Gabbay & Woods (2003) and Woods (2013, chapter 2).

³⁷Aristotle makes this claim in only two places, *On Interpretation*, 17a, 13 and 19 ff, 24, and nowhere mounts a defence of it. Here is Robin Smith on this point: “...since Aristotle thought that all propositions could be analyzed as categoricals, he regarded the syllogistic as the theory of validity in general.” (Smith, 1995, p. 35). I agree with Smith with regard to the reduction claim, but part company from him as regards “validity in general”. There is no such theory in Aristotle’s early writings.

³⁸The premiss consistency requirement comes about in two ways. First, Aristotle allows for a syllogisticity-preserving operation - let’s call it argumental contraposition - ac-

ording to which any syllogism $\langle A, B, C \rangle$ has an equivalent contrapose $\langle A, \text{not-}C, \text{not-}B \rangle$. Consider now the circular argument $\langle A, B, A \rangle$. Clearly not a syllogism, neither is its contrapose $\langle A, \text{not-}A, \text{not-}B \rangle$. The second source of the premiss consistency requirement is that any argument whose premisses are one another's contraries or subcontraries will fail the three-terms condition on syllogisms.

³⁹The theorem *ex falso quodlibet* asserts that a contradiction implies any statement whatever. In a paraconsistent logic, inconsistent premisses entail only proper subsets of them. In a syllogistic logic, the question of omniderivability doesn't arise. Syllogistic logics refuse admission to inconsistent premisses. The requirement that there not be multiple conclusions is an adumbration of one aspect of intuitionism.

⁴⁰More accurately, some writers see paraconsistency as the failure of the non-contradiction principle to be a theorem. Many others see it as the failure of contradictions to trivialize. For others still, paraconsistency is the failure of some inconsistent premiss sets to trivialize. But the dominant opinion is that the failure of *ex falso* is necessary and sufficient for paraconsistency. See Brown (2007, p. 97, n. 3).

⁴¹"No S is P" is recastable as "All S is not-P".

⁴²An anticipation of (Quine 1948).

⁴³Given the support of the perfectability proof of the *Prior Analytics*. See again Corcoran (1972) and also his (1974). As noted, an argument is a perfect syllogism if and only if it is obviously a syllogism. An argument is an imperfect syllogism if and only if it is a syllogism but not obviously so, whose conclusion is derivable by the law of contradiction, the laws of propositional conversion, the *reductio* law and further derivation rules that are obviously, i.e. perfectly, syllogistically valid. Some of the perfectability rules are clearly not syllogistic rules. Wouldn't this violate the requirement that deductive reasoning be syllogistic? No. There is no such requirement. For, recall, syllogisms were purpose-built for the resolution of *dialectical* wrangles. Even so, the perfectability proofs contain premisses which clearly are not categorical propositions. Doesn't this contradict the categorical reduction thesis? Not knowing its proof, it is hard to say. If there were a successful proof, there would be room to allow the on-sufferance use of premisses in non-categorical form. But the likelier option is that the thesis is so deeply untrue that Aristotle is forced to override it at the apex of his achievement of logic's first logical breakthrough.

⁴⁴See here Marion & Rückert (forthcoming).

⁴⁵Let me note in passing a more recent case in point, and a more contentious one. FDE, the logic of first degree entailment is a logic of relevance Routley & Routley (1985). In the semantics of FDE "not A" is true just in case A is, not false, but *possibly* false. FDE has a large fan-base. Many serious-minded logicians think that FDE gets "consequence" right or at least that its provisions for "consequence" are strongly supportable. But hardly any of these same logicians think that FDE gets "not" right. They think that there is no sense of negation which their "not" actually captures. Instead, they think that in getting consequence right they got negation wrong, on purpose so to speak. Thus many of FDE's supporters concede that FDE lacks an operator that is *recognizable* as negation an exception is Restall (1999). But not even its detractors say, or need say, that this makes FDE's consequence relation equally unrecognizable. What this suggests is that it is not the business of logic to get "not" right. Its business is to give to "not" a treatment that facilitates its getting "consequence" right. That is, it is logic's business not to get "not" so wrong that it can't get "consequence" right. The idea of a would-be logic's making

unrecognizable its purported notion of negation arises with Quine's dismissive discussion of dialethic logic and its espousal of "an occasional sentence and its negation as true". (Quine, 1970/1986, p. 81).

References

- Anderson, Alan R. & Belnap, Nuel D. 1975. *Entailment: The Logic of Relevance and Necessity, Vol. I*. Princeton University Press.
- Aphrodisias, Alexander. 1881a. 'Commentaria in Aristotelem Graeca, volume 2, part II'. In M. Wallies (ed.) 'Aristotelis Topicorum Libros Commentaria', Berlin: Reimer.
- . 1881b. 'Commentaria in Aristotelem Graeca, volume 2, part I'. In M. Wallies (ed.) 'Aristotelis Analyticorum Priorum Librum I Commentarium', Berlin: Reimer.
- Baird, Douglas G., Gertner, Robert H. & Picker, Randal C. 1998. *Game theory and the law*. Harvard University Press.
- Barth, Else M & Krabbe, Erik C. 1982. *From Axiom to Dialogue: A philosophical study of logics and argumentation*. Walter de Gruyter.
- Binnmore, Ken. 2005. *Natural justice*. Oxford University Press.
- . 2008. *Rational decisions*. Princeton University Press.
- Bochenski, Joseph M. 1970. *A history of formal logic*. New York: Chelsea.
- Boger, George. 2004. 'Aristotle's underlying logic'. In Dov M. Gabbay and John Woods, (eds.), *Greek, Indian and Arabic Logic*, volume 1 of Gabbay and Woods, (eds.), *Handbook of the History of Logic*, Amsterdam: North-Holland, (2004 ff).
- Brown, Bryson. 2007. 'Preservationism: A short history'. In Dov M. Gabbay and John Woods, (eds.), *The Many Valued and Nonmonotonic Turn in Logic*, volume 8 of *Handbook of the History of Logic*, 95-127. Amsterdam: North-Holland.
- Corcoran, John. 1972. 'Completeness of an ancient logic'. *The journal of symbolic logic* 37, no. 4: 696-702.
- . 1974. 'Aristotle's natural deduction system'. In 'Ancient logic and its modern interpretations', 85-131. Dordrecht: Reidel.
- d'Avila Garcez, Artur, Barringer, Howard, Gabbay, Dov M. & Woods, John. to appear. *Neuro-Fuzzy Argumentation Networks*. Amsterdam: Springer.
- Denyer, Nicholas. 1983. 'The origins of justice'. *Syzetisis: studi sull'Epicureismo Greco e Romano offerti a Marcello Gigante*. Naples 133-152.
- Dixit, Avinash K., Skeath, Susan & Reiley, David H. 1999. *Games of strategy*. Norton New York.
- Dutilh Novaes, Catarina. 2007. *Formalizing medieval logical theories. Suppositio, Consequentiae and Obligationes*. Dordrecht: Springer.
- Fudenberg, Drew & Tirole, Jean. 1991. *Game theory*. MIT Press.
- Gabbay, Dov M. & Woods, John. 2003. 'Normative models of rational agency: the theoretical disutility of certain approaches'. *Logic Journal of IGPL* 11, no. 6: 597-613.
- Gale, David & Stewart, Frank M. 1953. 'Infinite games with perfect information'. In H.E. Kuhn and A.W. Tucker, (eds.), *Contributions to the Theory of Games II*, *Annals of Mathematics Studies* 28, 245-266. Princeton: Princeton University Press.
- Gintis, Herbert. 2000. *Game theory evolving: A problem-centered introduction to modeling strategic interaction*. Princeton University Press.

- . 2009. *The bounds of reason: game theory and the unification of the behavioral sciences*. Princeton University Press.
- Hamblin, Charles L. 1970. *Fallacies*. London: Methuen.
- Harman, Gilbert. 1970. 'Induction: A discussion of the relevance of the theory of knowledge to the theory of induction (with a digression to the effect that neither deductive logic nor the probability calculus has anything to do with inference)'. In Marshall Swain, (ed.), *Induction, Acceptance and Rational Belief*, 83-99, Dordrecht: Reidel.
- . 1972. 'Is modal logic logic?' *Philosophia* 2, no. 1: 75-84.
- Henkin, Leon. 1961. 'Some remarks on infinitely long formulas'. *Infinitistic methods* 167-183.
- Hintikka, Jaakko. 1968. 'Language-games for quantifiers'. In Nicholas Rescher (ed.) 'Studies in logical theory', 46-72. Oxford: Blackwell.
- . 1973. *Logic, language-games and information: Kantian themes in the philosophy of logic*. Clarendon Press Oxford.
- . 1981. 'The logic of information-seeking dialogues: A model'. In Werner Becker and Wilhelm K. Essler (eds.) *Konzepte der Dialektik*, 121-131, Frankfurt am Main: Vittorio Klostermann.
- . 1987. 'The fallacy of fallacies'. *Argumentation* 1, no. 3: 211-238.
- . 1989. 'The role of logic in argumentation'. *The Monist* 72, no. 1: 3-24.
- . 1996. *The principles of mathematics revisited*. Cambridge University Press.
- . 1997. 'What was Aristotle doing in his early logic, anyway? A reply to Woods and Hansen'. *Synthese* 113, no. 2: 241-249.
- Hitchcock, David. 2000. 'Fallacies and formal logic in Aristotle'. *History and Philosophy of Logic* 21, no. 3: 207-221.
- Hodges, Wilfrid. 1983. 'Elementary predicate logic'. In Franz Guenther & Dov Gabbay (eds.) 'Handbook of philosophical logic', 1-129. Dordrecht: Reidel.
- . 1985. *Building models by games*. Cambridge: Cambridge University Press.
- . 2001. 'Dialogue Foundations: A Sceptical Look: Wilfrid Hodges'. In 'Aristotelian Society Supplementary Volume', vol. 75, 17-32.
- . 2004. "'Logic and games'". In Edward Zalta, (ed.) *The Stanford Encyclopedia of Philosophy*, <http://plato.stanford.edu/archives/win2004/entries/logic-games/>.
- . 2006. 'The logic of quantifiers'. In R.E. Audier & L.E. Hahn (eds.) 'The Philosophy of Jaakko Hintikka', 521-534. Chicago and La Salle: Open Court.
- Johnson, Ralph H. 1996. *The rise of informal logic: essays on argumentation, critical thinking, reasoning, and politics*, vol. 2. Vale Press.
- Kagel, John Henry & Roth, Alvin E. 1995. *The handbook of experimental economics*. Princeton University Press.
- Krabbe, E. C. W. 2001. 'Dialogue foundations: Part II: Dialogue logic restituted'. [title misprinted as "Dialogue logic revisited"], *Proceedings of the Aristotelian Society*, supplementary volume 75, 33-49.
- . 2007. 'Dialogue logic'. In Dov M. Gabbay and John Woods (eds.) *Logic and Modalities in the Twentieth Century*, volume 7 of *Handbook of the History of Logic*, 665-704, Amsterdam: North-Holland.
- Kuhn, Harold W. 1997. *Classics in game theory*. Princeton University Press.
- LaCasse, C. & Ross, D. 1994. 'The microeconomic interpretation of games'. *PSA 1994 volume 1*, 387-479, D. Hull, S. Forbes and R. Burien, (eds.). East Lansing, MI: Philosophy of Science Association.

- Lear, Jonathan. 1980. *Aristotle and logical theory*. Cambridge: Cambridge University Press.
- Locke, John. 1970. *An essay concerning human understanding*. Oxford: Clarendon Press. First official date of publication 1690.
- Lorenz, Kuno. 1961. 'Arithmetik und Logik als Spiele'. PhD Dissertation, Christian-Albrechts-Universität, Kiel, 1961. Selections reprinted in Lorenzen and Lorenz, 17-95.
- Lorenzen, Paul. 1960. 'Logik und agon'. *Atti Congr. Internat. di Filosofia* 4: 187-194.
- Lorenzen, Paul & Lorenz, Kuno. 1978. *Dialogische logik*. Wissenschaftliche Buchgesellschaft Darmstadt.
- Lukasiewicz, Jan. 1957. *Aristotle's syllogistic from the standpoint of modern formal logic, 2nd edition*. Oxford: Clarendon Press.
- Marion, Mathieu. 2009. 'Why Play Logical Games?' In Ondrej Mayer, Ahti-Veikko Pietarinen & Tero Tulenheimo (eds.) 'Games: Unifying Logic, Language, and Philosophy', 3-26. Springer.
- . 2013. "'Game semantics and the history of logic: The case of Greek dialectics'". *This Yearbook*.
- Marion, Mathieu & Rückert, Helge. forthcoming. "'Aristotle on universal quantification'".
- Maynard Smith, John. 1982. *Evolution and the Theory of Games*. Cambridge: Cambridge University Press.
- McCall, Storrs. 1963. *Aristotle's modal syllogisms*. North-Holland Amsterdam.
- McKirahan, Richard D. 1992. *Principles and Proofs: Aristotle's theory of demonstrative science*. Princeton University Press Princeton.
- Montague, P Read & Berns, Gregory S. 2002. 'Neural economics and the biological substrates of valuation'. *Neuron* 36, no. 2: 265-284.
- Nash, John. 1950a. 'The bargaining problem'. *Econometrica* 18: 155-162.
- . 1950b. 'Equilibrium points in n-person games'. *Proceedings of the national academy of sciences* 36, no. 1: 48-49.
- . 1951. 'Non-cooperative games'. *The Annals of Mathematics* 54, no. 2: 286-295.
- Otto, Martin. 1997. *Bounded Variable Logics and Counting. A study in Finite Models*. Springer-Verlag.
- Patterson, Richard. 1995. *Aristotle's Modal Logic*. Cambridge University Press.
- Patzig, Günther. 1968. *Aristotle's Theory of the Syllogism*. Dordrecht: Reidel.
- Pauly, Marc & Parikh, Rohit. 2003. 'Game Logic'. *Studia Logica* 72: 163-256.
- Peters, Stanley & Westerståhl, Dag. 2006. *Quantifiers in language and logic*. Oxford University Press.
- Poundstone, William. 1992. *Prisoner's Dilemma*. Doubleday, New York.
- Quine, Willard V. 1948. 'On what there is'. *The Review of Metaphysics* 2: 21-38.
- . 1950/1982. *Methods of logic*, New York: Holt, Rinehart and Winston. Revised edition, Harvard University Press.
- . 1970/1986. *Philosophy of logic, 2nd edition*. Harvard University Press. First published by Prentice-Hall in 1970.
- Rahman, Shahid & Rückert, Helge. 2001. 'New Perspectives in Dialogical Logic'. A special issue of *Synthese*, 78.
- Rahman, Shahid & Tulenheimo, Tero. 2009. 'From games to dialogues and back'. In O. Mejer, A. Pietarinen & T. Tulenheimo (eds.) 'Games: Unifying Logic, Language, and Philosophy', 153-208. Dordrecht: Springer.
- Rahwan, Iyad & Simari, Guillermo R. 2009. *Argumentation in artificial intelligence*.

- Berlin: Springer.
- Restall, Greg. 1999. 'Negation in Relevant Logics (How I stopped worrying and learned to love the Routley star)'. In Dov M. Gabbay & Hans Wansing (eds.) 'What is Negation?', 53–76. Kluwer.
- Robinson, Richard. 1953. *Plato's earlier dialectic*. Oxford University Press.
- Ross, Don. 2005. *Economic theory and cognitive science: Microexplanation*. MIT Press.
- Routley, Richard & Routley, Valerie. 1985. 'Negation And Contradiction.' *Revista colombiana de matematicas* 19: 201–232.
- Schelling, Thomas. 1960. *The strategy of conflict*. Cambridge, MA: Harvard University Press.
- . 1984. *Choice and consequence*. Cambridge, MA: Harvard University Press.
- . 2006. *Strategies of Commitment: And Other Essays*. Cambridge, MA: Harvard University Press.
- Shoemith, David J. & Smiley, Timothy John. 1978. *Multiple Conclusion Logic*. Cambridge: Cambridge University Press.
- Skyrms, Brian. 1996. *Evolution of the social contract*. Cambridge University Press.
- Smith, Robin. 1995. 'Logic'. In Jonathan Barnes (ed.) 'The Cambridge Companion to Aristotle', 27–65. Cambridge University Press.
- Tennant, Neil. 1979. 'Language games and intuitionism'. *Synthese* 42, no. 2: 297–314.
- Thom, Paul. 1981. *The syllogism*. Philosophia Munich.
- Tuozzo, Thomas. 2011. *Plato's Charmides: Positive Elenchos in a "Socratic" Dialogue*. Cambridge: Cambridge University Press.
- van Benthem, Johan. 2011. *Logical dynamics of information and interaction*. Cambridge University Press.
- van Benthem, Johan, Gupta, Amitabh & Pacuit, Eric. 2011. *Games, norms and reasons: logic at the crossroads*. Dordrecht: Springer.
- Vlastos, Gregory. 1982. "The Socratic elenchus: method is all". With an appendix and postscript, in Gregory Vlastos, *Socratic Studies*, Myles Burnyeat, (ed.), 1-37, Cambridge: Cambridge University Press.
- von Neumann, John & Morgenstern, Oskar. 1944. *The theory of games and economic behavior*. Princeton: Princeton University Press.
- von Plato, Jan. 2013. *Elements of Logical Reasoning*. Cambridge University Press.
- Waismann, Friederich. 1967/1979. 'Wittgenstein and the Vienna Circle: Conversations Recorded'. Brian McGuinness, (ed.) and J. Schutte and Brian McGuinness, translators. Oxford: Blackwell, 1967. Reprinted in 1979.
- Weibull, Jörgen W. 1995. *Evolutionary game theory*. Cambridge: MIT press.
- Wittgenstein, Ludwig. 1953. 'Philosophical Investigations'. G.E.M. Anscombe and Rush Rhees, (eds.) and G.E.M. Anscombe, translator. Oxford: Blackwell.
- Woods, John. 2001. *Aristotle's earlier logic*. Oxford: Hermes Science.
- . 2012. "A history of the fallacies in western logic". In Dov M. Gabbay, Francis Jeffrey Pelletier and John Woods, (eds.), *Logic: A History of its Central Concepts*, volume 11 of *Handbook of the History of Logic*, 513-610. Amsterdam: North-Holland.
- . 2013. *Errors of Reasoning: Naturalizing the Logic of Inference*. London: College Publications.
- Woods, John & Hansen, Hans. 1997. 'Hintikka on Aristotle's fallacies'. *Synthese* 113, no. 2: 217–239.
- . 2004. 'The subtleties of Aristotle's non-cause'. *Logique et Analyse* 176: 395–415.

- Woods, John & Irvine, Andrew. 2004. "Aristotle's early logic". In Dov M. Gabbay and John Woods, (eds.), *Greek, Indian and Arabic Logic*, volume 1 of *Handbook of the History of Logic*, 27-99, Amsterdam: North-Holland.
- Woods, John & Walton, Douglas. 1989/2007. 'Fallacies: Selected Papers 1972-1982'. 2nd edition with a Foreword by Dale Jacquette. London: College Publications. First edition Dordrecht: Foris, 1989.
- Yrjönsuuri, Mikko. 1994. *Obligations: 14th century logic of disputational duties*, vol. 55. Helsinki: Acta Philosophica Fennica 55.