Mathematics by Design

Introduction

Imagine a mathematics classroom where students eagerly arrive and do not want to leave, where students help one another, where deep understanding of mathematical concepts occurs, verified by multiple assessments, and where multiple modes of learning are designed into learning experiences. We were able to turn this imagined classroom into reality. How did we do it? Christine Krowles is a fifth grade mathematics teacher and Dave Burghardt a university professor. They co-designed the unit, Christine implemented the unit, Dave periodically visited the class, and they collaborated on the written analysis.

The students, working in teams of two, were challenged at the commencement of this unit to design and construct a chair for a stuffed animal they have been shown and the chair had to meet a variety of geometric specifications. They were enthusiastic about constructing the chair. The rationale for choosing a chair is that a chair can mean different things to different people, it may be a functional object only that is stiff and rigid, or comfortable and soft, there is no correct answer. Students do not have to learn about chairs, they have that knowledge, and they can apply mathematical principals to their chair designs. It is here that the unit makes a connection with the students' own lived experiences. The students were excited to construct their designs, but first they had to demonstrate their designs included all the geometric elements required. Therefore, learning the mathematical concepts became important to them.

In order to assess student knowledge improvement in geometry, it was necessary to determine the baseline of students' geometric knowledge before the unit began. To assess each student's level in the content domain, a written pretest was given to the students before beginning the unit. The grades ranged from 0% to 40%. The average score for the pretest was 17.6%, while the median score and the score earned most often was a grade of 16%. A written posttest, identical to the pretest, was given. The results were dramatically different. No students failed the exam. The lowest grade was a 76%, the average was an 88.6%, the median was 87%, and the grade earned most often was 96%. Every student increased by at least 40 points upon their own pretest score and most students improved by 80 to 99 points.

There were equally dramatic improvements in student attitude towards mathematics. We had anticipated that using design pedagogy would improve student learning, but not this dramatically. As is indicated by the low pretest scores, the students in Christine's class were homogeneously grouped by prior low mathematics performance in fourth grade.

This article addresses the results of an action research project in a fifth grade elementary school classroom. The research was conducted as part of an M.A. Program in Elementary Education with a specialization in Mathematics, Science and Technology (MST) designed for experienced elementary school teachers who seek the skills and knowledge, and attitudes and dispositions to integrate the teaching of these areas (Koch & Burghardt, 2002).

Research

In this action research project, there were content-based goals related to mathematics and to technology education consistent with the Geometry Standards for Grades 3-5 as prescribed by the National Council of Teachers of Mathematics (NCTM, 2000) and the Standards for Technological Literacy (ITEA, 2000). Geometry is a topic that is often difficult for students to grasp at the fifth grade level, particularly for students who have been placed by their district in the lowest of its three homogeneous math classifications.

In recent years, there has been a growing recognition of the educational value of design activities in which students create external artifacts that they share and discuss with others [Soloway, 1994; Papert, 1993; Resnick, 1998]. A synthesis of the literature reveals that pedagogically solid design projects involve authentic, hands-on tasks; use familiar and easy-to-work materials; possess clearly defined outcomes that allow for multiple solutions; promote student-centered, collaborative work and higher order thinking; allow for multiple design iterations to improve the product; and have clear links to a limited number of science and engineering concepts [Crismond, 1997]. The value of design-based activities in mathematics has shown that students who were not high achievers in traditional mathematics, were very successful when using design [Weideman & Hunt, 1997]. However, in classroom settings problems are usually well defined, so students have little experience with open-ended problems inherent to technological design problems which are seldom well defined. The design process begins with broad ideas and concepts and continues in the direction of ever-increasing detail, resulting in an acceptable solution [Thacher, 1989]. So using design in the classroom can be challenging as students are not familiar, or initially not comfortable, with the open-ended nature of design.

Many educators discuss the integration across disciplines but at times the standards based movement forces us to be more discipline focused. “No matter what the content, we can design active linkages between fields of knowledge. An interdisciplinary approach to learning may be seen as a curriculum approach that consciously applies methodology and language for more than one discipline to examine a central theme, problem or experience” (Jacobs, 1989). Research addresses the importance of hands-on activities, which, supported by meaningful discussion and theory building (Brooks and Brooks, 1993) help students construct meaning. Further, when students are encouraged to create artifacts (Appleton, 2000), they both reflect and enhance student understanding.

The particular design strategy used was based on the informed design cycle (Burghardt and Hacker, 2003). It is iterative and allows, even encourages, users to revisit earlier assumptions and findings as they proceed. Figure 1 shows the overall cycle. A key differentiating factor in the informed design process is in the Research and Investigation phase. The use of Knowledge and Skill Builders (KSBs) provides structured research in key ideas that underpin the design solution. The KSBs are short, focused activities designed to help students identify the variables that affect the performance of the design.

They provide structured research in key technology, science, mathematics processes, skills, and concepts that underpin the design solution. They also provide evidence upon which teachers can assess student understanding of important ideas and skills. The final design is “informed” by the knowledge and skills that students acquired in order to design and construct their solutions and relied directly to the knowledge gained from these KSBs.

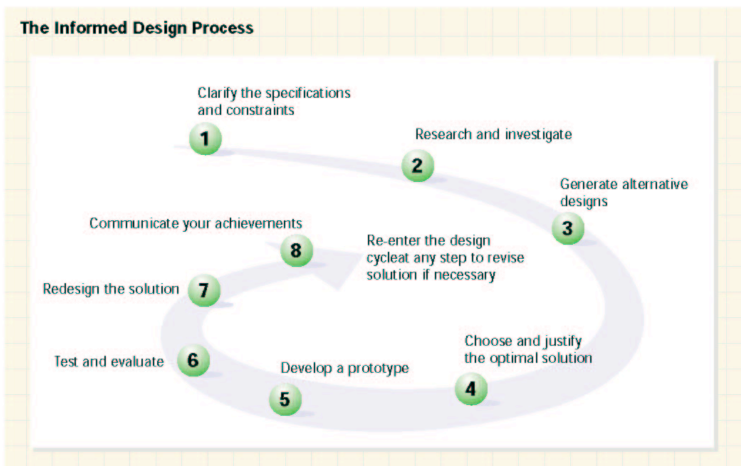

Figure 1: The Informed Design Cycle

Dave created and teaches a graduate course, Technology Education in the Elementary School, as part of the MA/MST program, which provides teachers with an understanding of the pedagogical and procedural aspects of the informed design process. Christine was a student in this class, prior to starting her action research project as part her master’s thesis. It is here that she learned about the informed design cycle as well as the importance of KSBs in a design-based project. The class was also instrumental in inspiring Christine, along with her colleagues, and demonstrating that design technology is a very powerful tool in motivating students and driving them to learn more about virtually any topic. This point became more clear as this particular MST unit progressed.

Unit Goals

There were certain goals to be reached by implementing this unit. The students should:

- learn how to use a protractor to both measure and create angles of a given measure
- be able to identify and create acute, obtuse, and right angles
- be able to classify and create scalene, isosceles, and equilateral triangles
- be able to identify and create the different types of quadrilaterals including trapezoids, rhombuses, parallelograms, squares, and rectangles
- have working knowledge of similarity and congruency in polygons

The chair design specifications help ensure the assessment of student geometrical understanding. For instance, the chair must include within it at least:

- one right angle
- one acute angle
- one obtuse angle
- one isosceles triangle
- one scalene triangle
- one trapezoid with an angle that measures 55 degrees
- two congruent rhombuses
- two similar parallelograms

The students could choose how and where within the design they will incorporate these elements. However, they had to provide a labeled drawing with the finished product that showed where each element is located within the chair.

Implementation of Unit

The following comments from Christine's teaching journal show that the class initially was enthusiastic about designing a chair.

> "I began the unit today by discussing the project with the class. I have been telling them little bits and pieces of the project before today, just to get them excited. However today I really told them about the project in depth. I gave them the handout that describes the project and we talked about it. I was really surprised by how excited they actually got. So far this year, the class has been anything but animated. I have been frustrated up until this point by how lifeless they seem no matter how hard I have been trying to get them stimulated. Today they seem really eager for the first time this year!
>
> We also discussed the materials that they want in order to make the chairs. Again, they were bursting with ideas. One of the groups was already talking about making a reclining chair. That has some tremendous possibilities in terms of the geometric polygons that could be incorporated into that.
>
> Finally, I gave them the attitude survey and the pretest. I gave the survey first and was surprised by how many kids were struggling to think of jobs that required math. As for the pretest (which will serve as the written test at the end of the

> unit), some of them just stared at it. I would imagine from their reactions that if they answered anything at all, it would be just guesses."

As the unit progressed, student attitudes towards mathematics and learning were substantially improved. A strategy that Christine employed as part of the design process was to have students decide on their best design (drawing of it) and then modify that as they learned new information. The students maintained design journals that contained their mathematics notes. The value of iteratively revising their design drawings prior to building the chair cannot be underestimated. Students willingly did this. There was a group decision as to where the geometric features would be added, then each student made changes in their individual journals.

However, there are concerns as well. Not all students were able to keep pace with each day's activities and Christine expressed concern that the unit was taking longer than anticipated. Her journal indicates on day 8,

> "I also am realizing the downfall of being so over prepared for a unit. As every day goes on, and I inevitably get to less and less (since I planned for much more than I can accomplish), I find that I am falling further and further behind. I have, until this point, made the mistake of continuing with the extra plans the following day, and so my unit is taking much longer to get through than I had thought it would in the beginning".

On day 9

> "Today the students did a KSB that I called "looking at chairs". Before the class even started, Alex asked me if we were working on our chairs today. The homework from last night was for the kids to bring in pictures of chairs that they cut out of magazines and newspapers. They then pasted them down to a worksheet and in the box next to the chair, there was a space for the students as a group to write what they liked and what they didn't like about the chair in question.
>
> It was really interesting to see how this activity got the students thinking about their own chairs that they will build and how the creativity started to show through. Even though this design challenge is not one where the students will all end up with the same product, I began to see today how the products really are going to be different. I spent a considerable amount of the period sitting with Larry, John, and Anthony's group. At first, this was the group that I was the most concerned about in terms of effort. I was really afraid that this group would slap some glue and sticks together and call it a day. Instead, I was amazed at the creative ideas that this group wanted to incorporate within their design. First, they decided that they wanted to build a chair that has wheels. They felt that mobility was an important character for a chair to have."

It is often a challenge for the teacher to keep the students focused on academic content when they want to pay attention to their designs. The strategy that evolved was to allow students to modify their design sketches after they demonstrated they understood new mathematical concepts. Christine noted on day 16 that "I allowed the class to use the extra time to work on their designs with their groups. Even so, I am noticing reluctance among all of my students to leave the class when the period is over. This especially exists among Nina, Gabe, Alex, and Emily (initially some of least proficient math students). I am pleased with this. Gabe stays after class every day to come up to me and talk to me about his chair."

The initial concerns about time were mitigated, though not eliminated, as the unit progressed. Because students were more attentive and focused on mathematics, later lessons were accomplished more quickly, leaving time for design updates. This is demonstrated by Christine's observations on day 17.

> "Again today I noticed that Alex really seemed happy in class. I wonder if I'm noticing this merely because it is such a sharp contrast to the beginning of the year when she sat with her head down every day. The first few weeks of school she insisted on sitting in the back of the room, even though there were rows of empty seats in front of her. Now, she looks so happy to be in class, it is hard to believe that this is the same student." On day 20, "Nina made an interesting comment to me today. She said that she thought that ever since we started the geometry unit, the period seems to go by so much faster than it did before and faster than the other periods of the day. I was pleased to hear this because it indicated to me that Nina is enjoying math more than she did according when she took her survey at the beginning of the unit."

Students demonstrated increased confidence in their mathematical reasoning, strongly defending their positions at times. For instance on day 24 Christine noted,

> "I've also been noticing an interesting trend in Gabe and Nina's group. As the unit has gone on, and the two of them have been showing me more and more confidence in math, they have been fighting a whole lot more. Originally, I had some reservations about placing them in a group together because they both had extremely low math confidence according to the math surveys that I gave at the beginning of the unit. However I did it anyway because Gabe had so few friends in the class that I thought that maybe by putting him in a group with his only friend might help his confidence a little. Now, the two of them argue over everything, each of them insisting to the other that they know exactly what should be done. The arguing that seems to take place always results from each of them trying to tell the other what should be done mathematically. Today's argument, for example, was about Gabe insisting that the two congruent rhombuses that Nina drew were not exactly congruent, and Nina arguing that they were. To me, whether the rhombuses were correct or not was less interesting than the idea that both of them were confident enough in their definitions of congruency to start an argument with a friend. This was an important cue to the growth of math confidence in both Nina and Gabe since the unit began."

After all of the topics had been taught and the students had created a final sketch that included all of the specifications, they started constructing their chairs. Again problemsolving skills were used. Students tried to figure out how to make the back of the chair stand up without tipping the chair over. When testing their designs, groups realized where supports needed to be added and tried to figure out what supports would be best. The math measurement standards were being addressed. Students had to decide whether they should use a protractor or a ruler to measure the geometric figures during construction. Should they use centimeters or inches? Most importantly, the students began to think mathematically while accomplishing their tasks. One group came to realize that even though their scalene triangle did not look scalene, it was the measurements that counted and not their visual perceptions of the triangle.

Results—Content Goals

As noted in the introduction, and shown graphically in Figure 2 , the pre/post test results indicated a dramatic improvement in mathematical understanding. This was confirmed by other assessments as well.

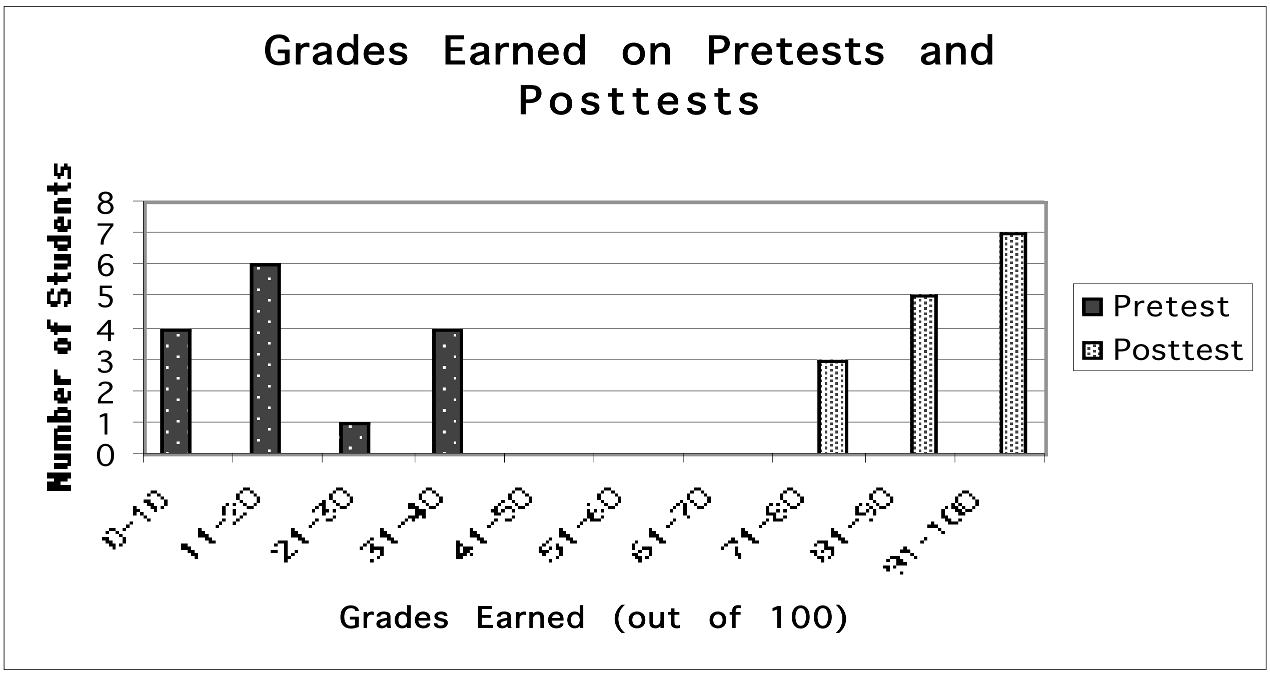

Figure 2 Comparison of Grades Earned on Pretests and Posttests

Ten out of the fifteen students, were able to identify at least 7 out of the 8 figures that were given on the exam. The other five students were able to identify 6 out of the 8 figures given on the exam. Most of the students in the class were able to raise their level of geometric thought by mastering the first or visual level on van Hiele's (1999) geometric reasoning scale.

Pierre and Dina van Hiele investigated different levels of geometric reasoning and determined five different levels of geometric understanding, ranging from the visualization level (level zero) to the deductive axiomatic level (level four). Although the levels are not necessarily age related, the first three levels (visualization, analysis, and informal deduction) are appropriate for elementary school students. At the visualization level, students recognize shapes by their appearance. Many of the students in Christine's class could not do this at the beginning of the unit, e.g. they could not distinguish a square from a rhombus, a rectangle from a parallelogram.

The next level, the analysis level, is characterized by awareness of the parts of figures, e.g. parallelograms have opposite parallel sides, but cannot explain relationships within a shape or between shapes. Most of Christine's students were able to achieve this level by the end of the unit, and some were able to achieve, or be close to achieving the third level, informal deduction. At this level, students can deduce properties of figures explain relationships within a figure and between figures.

After completing construction of the chairs, individual student interviews were held outside of the regular class time. The student was asked about each individual geometric specification that the chair was required to have and had to identify the item, such as a scalene triangle, in the chair and explain what they knew about the figure and prove it. A ruler and a protractor were sitting on the desk for them to use.

Students who received full credit were able to correctly discuss and prove the geometric properties of the polygon or concept (such as similarity or congruency) in order to prove its existence in the chair and therefore showed mastery of the analysis level. The learning demonstrated here was evaluated through a rubric that was created for this purpose shown in Table 1 (Steinkamp, 2002).

	Angles	Triangles	Trapezoid	Similar Parallelograms	Congruent Rhombuses
4	Student correctly identified all 3 angles (acute, obtuse, right) in design and can prove them.	Student correctly identified and proved both scalene and isosceles triangles.	Student identified trapezoid and was able to prove the 55 degree angle.	Student identified the similar parallelograms and could prove their similarity.	Student identified the congruent rhombuses and could prove their congruency.
3	Student correctly identified 2 angles in design and can prove them.	Student correctly identified both scalene and isosceles triangles but could not prove them.	Student identified trapezoid the 55 degree angle, but could not prove it.	Student identified the similar parallelograms and could explain their similarity, but not prove it.	Student identified the congruent rhombuses and could explain their congruency, but not prove it.
2	Student correctly identified 1 angle in design and proved it.	Student could correctly identify and prove one triangle. (other triangle may or may not be identified.	Student identified trapezoid, but could not find the 55 degree angle even with a protractor.	Student identified the similar parallelograms and could not explain or prove their similarity.	Student identified the congruent rhombuses and could not prove or explain their congruency.

1	Student could not identify or prove any angles.	Student could not correctly identify or prove either triangle.	Student could not identify trapezoid.	Student could not identify similar parallelograms.	Student could not identify congruent rhombuses.

Table 1 Practicum Rubrics for Assessing Geometric Knowledge

Seven students scored an 18 or a 19 on the interview and six students received a perfect score of a 20 on this assessment. This means that most of the class was able to identify every polygon and relationship that they were asked about. Further, they were able to explain the qualities of these polygons and use the ruler or protractor to prove that those qualities existed in the polygons they created. Therefore, thirteen of the fifteen students in the class were able to go on from first level to second level and were ready to begin work on the informal deduction level.

The students were also assessed for technological understanding using the design portfolio and the benchmarks shown in Table 2. Students completed the portfolios

Design Process
- Explained problem and identified constraints and specifications.
- Research and Investigation—for each Knowledge and Skill Builder.
- Sketched a variety of solutions, justified best design.

Design Solution
- Drew accurate sketch of final, as built, design.
- The solution worked. It met the design specifications and constraints.
- Aesthetics of design.

Mathematical Connections
- Presented results in graphs and charts.
- Accurate measurement and calculation.

Science Connections
- Tested the design.
- Provided conclusions based on testing and made recommendations for improvements.

Work Habits
- Worked collaboratively with classmates.
- Completed assigned tasks in a timely fashion.

Communication and Presentation
- Design portfolio completely and neatly accomplished.
- Actively participated in the presentation of results.

Table 2 Design Assessment Rubrics

for full credit, except for two students who had difficulty sketching. Students were asked as a homework assignment to reflect on their design experience. Three students wrote about things they learned through the process of physically building the chair indicating that they were surprised they could do so. One of these students wrote "that you can really build a chair if you try". Nine students responded to the question by writing about mathematics areas explored in the unit. A typical response was, "I learned that you could use right, acute, and obtuse angles when you make a chair". These students clearly believed that the project helped them learn about geometry. Four students wrote about how they learned about teamwork. The commonly wrote that they "learned how to work with a friend and put things together". Every student answered the reflection questions completely and this was a class that had great challenges in language arts, particularly writing. It was clear they spent a great deal of time thinking about and responding to these questions.

Regarding the *Standards for Technological Literacy* (ITEA, 2000), several benchmarks in Standards 3, 8, 9 and 11 were met. For example, from Standard 3, "Various relationships exist between technology and other fields of study (p 48)" and from Standard 9, the engineering design process involves defining a problem, generating ideas, selecting a solution, testing the solution(s), making the item, evaluating it, and presenting results (p102).

Results—Affective Goals

The affective goals, developing increased math confidence and fondness, were assessed with the same pre-survey that was given at the commencement of the unit. The survey began with eight statements, as shown in Table 3.

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. I am good at Math.				
2. I am better at Math than other subjects.				
3. I am better at Math than I was last year.				
4. I like Math.				
5. I like Math more than other subjects.				
6. I like Math more than I did last year.				
7. Doing Math can be fun.				
8. It is fun to learn new things in Math.				

Table 3 Fondness/Confidence Assessment

For each statement, the student had to decide if he or she strongly agreed, agreed, disagreed, or strongly disagreed. For the purposes of assessment of this data, a rating of -2 was assigned where the student strongly disagreed, a -1 was assigned where the student disagreed, a 1 was assigned where the student agreed and a 2 was assigned where the student strongly agreed to the given statement.

The first three statements dealt with the student's confidence in their own math abilities. Therefore, a score of -6 (-2 points per each of the three questions) would have indicated a student with very low confidence in their mathematic abilities and a 6 (2 points per question) would indicate a student with a great deal of confidence in themselves mathematically. The composite becomes their "confidence rating".

Figure 3 shows a comparison of these confidence ratings where the scores have been normalized to have values between 0 and 1. Visually, the farther apart the ends of the two bars are, the more improvement was shown in the child's self-confidence. There is a dramatic increase in confidence ratings of almost all of the students in the class. No students decreased, while three remained the same. The overall increase was 50.5 percent.

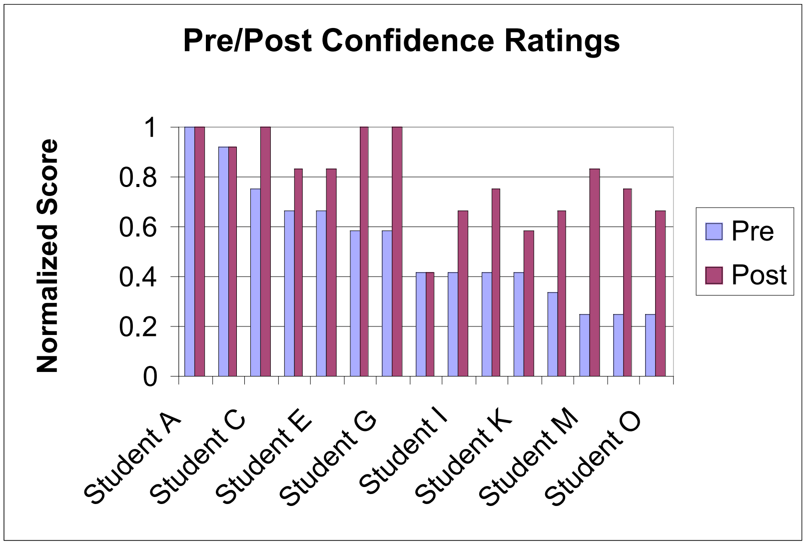

Figure 3 Pre/Post Assessment Confidence Ratings

The next five questions on the survey dealt with how much the student liked math. There were five questions, hence the lowest possible score would be a –10 indicating that the child did not like math; and the highest possible score would be a 10, indicating that the child did like math a great deal, creating a "math fondness rating". Figure 4 illustrates the comparison between these ratings found for each child on the pre-unit surveys and post-unit surveys where the scores have been normalized to have values between 0 and 1. All but two students showed improvement in their affection towards mathematics by the end of the unit, a dramatic change. The two students had an initially high fondness rating. No matter how the students felt about math before the unit began, the 63.6% increase showed that they liked it even more when the unit ended.

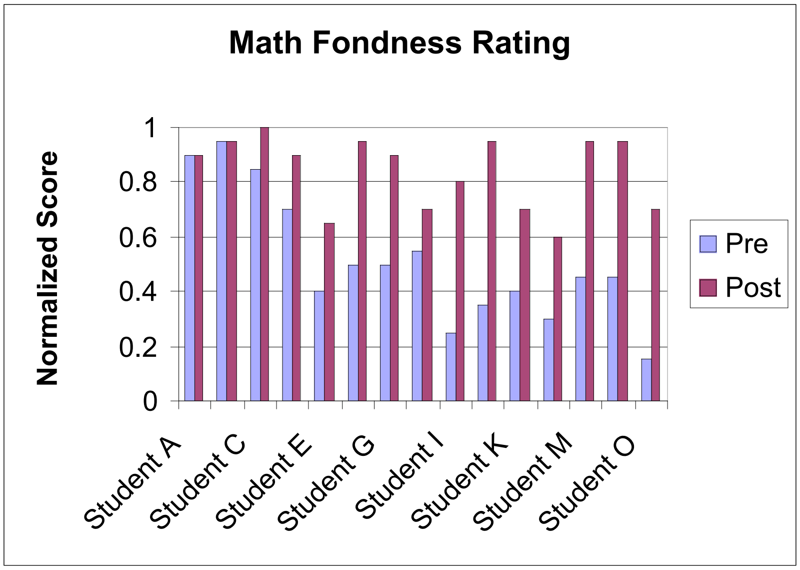

Figure 4 Pre/Post Assessment of Math Fondness

A teaching journal was kept throughout the duration of the unit. To quantify this qualitative data observation of three students' progress was made. The selected students indicated the lowest confidence and/or attitude on the pretest. A list was created of 5 behaviors that were felt to be indicative of math confidence, an interest in math, and a liking of math as shown in Table 4 (Steinkamp, 2002).

	Day 1	Day 5	Day 9	Day 13	Day 17	Day 21	Day 25	Day 29
Participated in class								
Asked questions								
Made verbal remarks indicating a liking, interest, or confidence in math								
Attentive in appearance (sitting up, attention focused on activity or lesson)								
Seems eager to be in math setting (arrives and is ready to work quickly, is reluctant to end activity or leave room), seems eager to work on projects, or visually appears happy to be in class.								
Total								

Table 4 Chart Used to Quantify Student Classroom Behavior

At the end of each designated day, the number of target behaviors that the child displayed was tallied for each student. At the end of the unit, the totals for each day could then be graphed as shown in Figure 5.

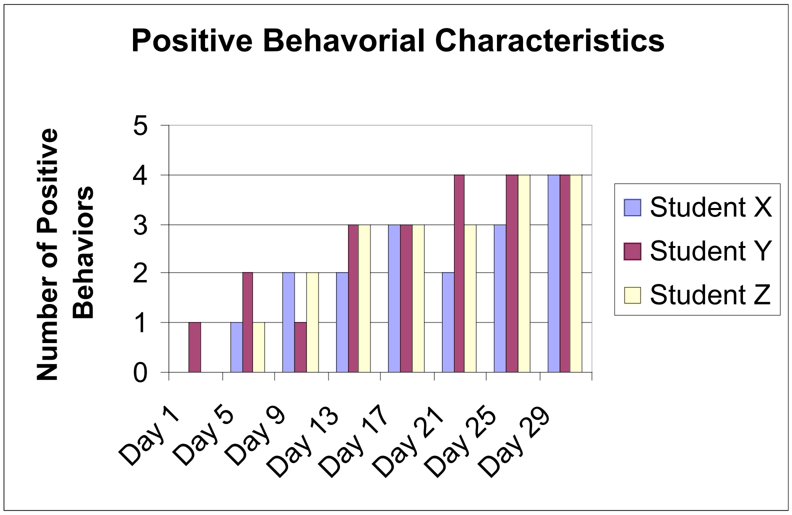

Figure 5 Positive Behavioral Characteristics of Selected Students

An increase was noted for each of the three students over the time period studied. All three students ended up showing four of the five possible positive behaviors by the end of the unit, indicating a dramatically positive change in attitude towards mathematics.

Also noted in the journal were the changes in the behaviors of all students as they entered and left the room at the completion of the unit as compared to when the unit first began. At the beginning of the unit, students often entered the class slowly and were hesitant to begin working. This reluctance was replaced with enthusiasm by the last days of the unit. Similarly, the same students who sped out of the classroom the second the bell rang were the same students who loudly groaned when it rang at the end of the unit.

Conclusions

Creating student-centered classrooms is often difficult in mathematics where a great deal of content must be dealt with. The use of design, while more time consuming initially, proved to be time-effective. Repetition of concepts was minimized, compared to previous students in prior years, because a deep sustained understanding had been created. From a teacher's perspective, the students continue to be interested in coming to class, are interested in new mathematics topics, and design continues to be an important strategy to help students with a wide range of needs, learn effectively.

Students increased their level of geometric understanding significantly. This understanding was verified through multiple means of assessment. Student attitude towards and confidence about mathematics also improved dramatically, as most of the students initially neither liked math nor felt confident about their mathematical abilities.

Design was used a pedagogical strategy throughout the unit, rather than as a culminating activity. Student attention was maintained throughout the unit as they updated their optimal designs based on newly learned material and revisited prior geometric knowledge in the process. Students liked the open-endedness of design, after initial concern that there was no one correct answer that would be provided by the teacher. Using a design project where the students knew the technology, e.g. how a chair is constructed, allowed focus on the geometric content. Minimal time was spent on fabrication techniques. While the focus of the unit was geometry, taught in a mathematics class, many of the benchmarks of the *Standards for Technological Literacy* (2000) were met.

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