



**An Online Journal for
Teacher Research**

An Action Research Project by Teacher Candidates and their Instructor into using Math Inquiry: Learning about Relations between Theory and Practice

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Abstract

This paper reports on what two teacher candidates and their instructor learned from an action research project into the use of inquiry to teach mathematics. We use a model of the relation between theory and practice in teacher education to interpret what we learned about inquiry. This model describes three modes for teacher candidates to learn about teaching: (1) applying theory to practice; (2) interpreting theory and practice, and (3) building/refining personal, practical and professional theories. We learned to (1) apply the 4D-Cycle Model of inquiry, (2) interpret what it means for inquiry to be flexible, and (3) build a theory of teaching with inquiry based on non-linear and community-based dispositions of teachers toward learning. We conclude by suggesting that this model could constitute a developmental pathway by teacher candidates for experiencing the linkages between theory and practice.

We are action researchers. Paul is an associate professor who teaches mathematics teaching methods to prospective K-8 teachers. Michelle and Krysta have completed their pre-service teacher education, but were teacher candidates at the time of the action research reported in this paper. We are passionate about teaching mathematics. Krysta and Michelle aspire to be dynamic and cutting edge teachers of mathematics. Paul encourages his students to take-up reform-based approaches to teaching mathematics.

In his second of two mathematics teaching courses in a two-year generalist elementary teacher education program, Paul used principles of action research to inscribe the major assignment for the course; an invitation for teacher candidates to become action researchers of the teaching of mathematics. When Michelle and Krysta took this course, they saw the assignment as an opportunity to conduct a collaborative action research

project on the use of inquiry learning in mathematics classrooms, which extended beyond the end of the course into their final teaching practicum. Paul participated in this project as both their teacher and mentor. This paper reports on what Krysta, Michelle, and Paul learned from participating in this action research project on inquiry.

Our action research led us into a consideration of possible linkages between the theory and practice of using inquiry learning in mathematics classrooms. The teacher education literature has clearly described the polarizing of theory and practice in the learning experiences of pre-service teachers (e. g., MacDonald, Falkenberg, & Goodnough, 2012). Pre-service teacher education programs, by their very nature tend to silo theoretical understandings from practical experience (Russell & Dillon, 2015; Zeichner, 2010). It is common in traditional teacher education programs to assume that teacher candidates apply the theory they learn in course work during their practicum experience (Russell & Dillon, 2015). For teacher candidates, the university is positioned as imparting theories of education, which may or may not be helpful as they learn how to teach in their practicum placements. The practice of teaching in schools is perceived as more useful by teacher candidates, and theory is only useful when it “applies” to practice. But the relationship between theory and practice is richer than linear and one-directional application of theory to practice (Betts & Block, 2013; Zeichner, 2010). A more nuanced understanding of the possible linkages between the theory and practice of inquiry-based teaching was triggered by our action research project.

Theoretical Framework for this Paper

We used three theories in this paper, namely, action research, inquiry learning, and the relation between theory and practice in teacher education. Action research was the methodology, inquiry learning was the math teaching pedagogy we sought to understand further, and the relation between theory and practice was the theoretical framework used to interpret what we learned from our action research project. In this section, we describe literature on each of these theories that are relevant to our project, and close by noting connections between these theories.

Action Research

Teacher education is shifting from transmission to situated and collaborative approaches to learning about teaching (Borko, 2004). Situated and collaborative approaches assume that learning about teaching should be grounded in and arising from the in-situ and contextual practice of teachers, and that professional learning communities support richer and critical learning experiences (Rigelman & Ruben, 2012). These situated and collaborative approaches are grounded in social constructivist theories of learning, such as Vygotsky (1978) and Lave and Wenger (1991), where learning is mediated by social artifacts, the learning milieu and interactions with others (whether they are novices, peers or experts).

Action research is one such collaborative and situated approach for designing learning experiences for teacher candidates. Several key principles of action research were used by Paul to design an assignment for an undergraduate course in elementary mathematics teaching methods. These key principles were that teacher candidates work together to engage with a problem of teaching emerging from their own practice, seeking to

explore the problem through cycles of reflection and action (McNiff & Whitehead, 2006). Ideally, a professional learning community (in this case, consisting of the instructor and two teacher candidates) would emerge where dialogue among participants was both supportive and critical (Betts, 2011; Darling, 2001).

More and more, teacher education programs in North America are building action research into their curriculum, either as part of course work or as an overarching goal of the program (Costello, 2011; Goodnough, Bullock, & Power, 2015). The programs at OISE and the University of Calgary are examples of the later, while the program at the University of Winnipeg uses the former approach. Action research is seen as a critical mechanism for fostering the beginning professional learning experiences of teacher candidates, and as a *modus operandi* of effective teachers (Levin & Rock, 2003). Action research fosters collaborative, reflective and in-situ experiences, which appear to be necessary conditions for exploring the complexities of teaching (Levin & Rock, 2003).

Within the mathematics education literature, there are a growing collection of studies concerning teacher's collaborative work, including action research. Goldsmith, Doerr and Lewis (2014), in their review of the literature, noted that participation in a learning community was needed for teachers to develop richer understandings of mathematics teaching. Jaworski (2006) argued that inquiry (another label for a teacher learning model similar to collaborative action research) in mathematics teaching was enhanced when there is a "critical alignment" between teacher's practice and critical reflection on math teaching possibilities. This critical alignment was a means to challenge the status quo in mathematics education, which avoided a reproduction of traditional teaching practices. Further, studies of mathematics teacher research that were grounded in practice tended to describe teacher learning as incremental, non-linear and iterative, as teachers' questions, actions and reflections tended to interweave (Goldsmith, et. al., 2014).

In a synthesis of the literature, Towers (2010) concluded that beginning teacher's knowledge of teaching and what they actually do were not necessarily aligned – they can "talk-the-talk" but not "walk the walk." In response to this disjunction between teacher knowledge and action, Towers found evidence that teacher candidates can learn to walk-the-walk within collaborative and action-based teacher education environments. In particular, Towers described a case study of a recent teacher education graduate who developed his ability to successfully enact an inquiry-based pedagogy of mathematics instruction. In contrast to traditional approaches to teacher education, action research can have an impact on prospective teacher's understanding of the theory of mathematics teaching, and its enactment in practice.

Finally, the potency of action research within pre-service teacher education programs seems to be in need of further development in the teacher education and teacher researcher literature in regards to the voices of teacher candidates. A scanning of the literature over the past 20 years bears this out. The literature includes many studies by in-service teachers of their own practice, by teacher educators of their own practice, and by teacher educators *about* pre-service and in-service teachers. But, a survey of the *Canadian Journal of Teacher Research*, *Journal of Teacher Education*, *Teacher Education Quarterly*, *Networks: An Online Journal of Teacher Research*, *Studying Teacher Education: A Journal of Self-Study of Teacher Education Practices*, and *Teaching and Teaching Education* resulted in

very few studies by pre-service teachers, chronicling what they learned while conducting action research projects. A recent special edition in *Networks: An On-line Journal for Teacher Research* focussed on pre-service teacher action research, and one article stated, "While much attention has been paid to action research conducted by in-service teachers, there appears to be an absence in the practitioner literature examining pre-service teacher's relationship with action research" (Fasching-Varner, Dowell, Meidl, & Meidl, 2013, p. 1). The time has come to pay more attention to the voices of pre-service teachers engaging in action research.

Inquiry Learning in Mathematics Classrooms

A common theme in theories of teaching is shifting from focusing solely on transmission to also incorporating facilitative teaching approaches. One such facilitative approach was based on inquiry learning, where the learning environment was designed so that students have more choice in what they will learn and how they will learn. The general assumption was that choice motivated and empowered students to learn, and that this learning environment reflected the learning abilities needed in the 21st century.

There have been several reviews of the literature that consider the effectiveness of inquiry learning (Friesen, & Scott, 2013; Furtak, Seidel, Iverson, & Briggs, 2012; Little, 2010), including challenges (Edelson, Gordin, & Pea, 1999) and critiques (Kirschner, Sweller, & Clark, 2006). According to a report published by the Canadian Education Association (2014), which summarizes this literature, inquiry learning was effective in various subjects in terms of both outcomes and larger abilities such as critical thinking, provided there was appropriate guidance from the teacher.

The National Council of Teachers of Mathematics (2011) advocated for the use of high-level tasks and expectations in the teaching of mathematics, which could be accomplished with an inquiry approach (Goos, 2004). Beyond the assumptions about inquiry noted in a previous paragraph, an inquiry learning approach in math was intended for students to experience how mathematicians work (Artigue, & Blomhøj, 2013). Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, and Wearne (1996) argued that inquiry in mathematics needed to be grounded in a fundamental aspect of mathematics, namely problem solving. In this approach, open-ended and rich mathematical tasks are designed to trigger and sustain the mathematical activity of students. Gravemeijer (1999) argued that the point of departure for an inquiry needed to be a realistic situation. In both approaches, the key pedagogical idea was to occasion deeper mathematical thinking by shifting from telling students what they should know about mathematics to designing learning environments where students could think like a mathematician.

Several pedagogical models have been developed to support teachers who use inquiry to design the learning environment in their mathematics classrooms (e.g., Allmond, Wells, & Makar, 2010; Smith, Bill, & Hughes, 2008; Van den Heuvel-Panhuizen, 2000). These models share three commonalities: (1) A rich, high-level and open-ended task is used, in which there are several possible types and levels of mathematics possible; (2) teachers are required to guide students through the task in ways that build on prior knowledge while also providing opportunities for students to recognize the deeper mathematics that they are learning (this learning may be outcomes such as fractions

and/or processes such as logical thinking); and (3) students have opportunities to share their thinking processes and products with others, including peers, teachers and parents. There is a complex relation between student's prior knowledge and the mathematics they are learning because the open-ended tasks used make possible connections between mathematical tools already learned and applied to the situation and developing not-yet-learned mathematical tools. This complex relation requires teachers to walk-the-line between telling students what to do and facilitating their thinking.

Relation between Theory and Practice in Teacher Education

The existence of a phenomenological gap between theory and practice, as experienced by pre-service and in-service teachers participating in various teacher education programs, is well-documented in the literature (MacDonald, Falkenberg, & Goodnough, 2012). Several approaches have been theorized for addressing a richer understanding of the relationship between theory and practice when learning to teach. Korthagen and his colleagues (Korthagen, 2001; Lunenberg, & Korthagen, 2009) proposed a distinction between theoretical knowledge, which was propositional in nature, and practical wisdom, which was not describable in words but emerged from connections between concrete present situations and prior experience. Russell and Dillon (2015) proposed a shift from theory-into-practice (the traditional assumption of teacher education) to practice-and-theory where it is assumed that theory and practice are dialectically related.

Some research has indirectly examined the theory-practice gap for mathematics teacher education. Using a focus on rich mathematical tasks, Slavit and Nelson (2010) found that, within a collaborative action research project, mathematics teachers were able to develop individual and collective theories of mathematics learning and instruction. This research points at a potential for action research to provide opportunities for teachers (pre-service or in-service) to learn about the linkages between theory and practice.

In this paper, we used a model for the relation between theory and practice first proposed by Paul and his colleague (2013). This model interrogated a linear and one-directional relationship between theory and practice in the context of general learning by teacher candidates in a teacher education program. Betts and Block (2013) proposed that there are three modes in which teacher candidates can experience a linkage between theory and practice. First, as commonly assumed, teacher candidates can apply theory to practice. Second, similar to the practice-and-theory perspective of Russell and Dillon (2015), teacher candidates can interpret theory and practice, which means that either a theoretical idea was used to interpret a teaching experience, or a practical experience was used to interpret an educational theory. Third, was building/refining a personal, practical and professional theory of teaching, where a teacher candidate developed a personal educational theory based on practical experience and understandings of educational theories, which was similar to the theory building described by Slavit and Nelson (2010).

Connecting the Theories

According to Darling-Hammond (2006), action research may mitigate against the siloing of theory and practice because teacher candidates have opportunities to critically examine educational theory based on their teaching experiences. Jaworski (2004) argued

that learning to use inquiry in math is a complex endeavor, requiring opportunities to examine practice within professional learning communities. We were able to add to the literature on math teaching inquiry by using action research. The cycles of action and reflection within a learning community (the three of us) allowed us to notice some of the complexity of inquiry. The model for the relation between theory and practice served as a framework for organizing, making sense of, and interpreting what we learned about inquiry during this action research project.

The Context for this Action Research Project

Krysta and Michelle were students in a final year mathematics teaching methods course, for which Paul was the instructor. The mathematics education professional learning assignment, built using principles of action research, was the only assignment for the course. In particular, students picked a mathematics education topic of relevance to their teaching practice within their practicum school, and work together to explore their topic of choice. Students were expected to link ideas from research with their teaching practice, to try out various math teaching ideas in their practicum, and to reflect on these experiences. The instructor, Paul, was available for guidance throughout the process.

Krysta and Michelle chose to study the use of inquiry as a method of teaching mathematics within grade six to nine classrooms. They reviewed the mathematics education literature concerning inquiry, generated several example inquiry lessons, and tried out some of the examples to inform their reflections concerning inquiry. A report on their findings up to the point of the course ending served as a final product for course assessment purposes, but the project did not end. Krysta and Michelle continued to develop and enact several math inquires during their final practicum block, collaborating with each other in their reflections on day-to-day aspects of their inquiry lessons. At the end of the practicum, Krysta, Michelle and Paul reflected on how their action research had proceeded during the course and practicum.

Krysta and Michelle's research into teaching mathematics with inquiry was initiated by an education course assignment, but the process continued into the practicum so it represents a legitimate experience of action research. The topic of inquiry was situated in their learning needs and their practice as emerging teachers. A professional learning community arose, where Krysta and Michelle explored a topic, seeking and accepting feedback from Paul and other educational professionals throughout the process.

We kept track of our learning throughout the process by making field notes of face-to-face conversations, saving e-mail dialogues, referring to the report that was the final product of the course assignment, and Krysta and Michelle wrote a final reflection concerning their perceptions of their inquiry lessons during the block. After the practicum and completion of the final reflection, Paul re-examined the data for themes in what Krysta and Michelle learned during their action research project. Paul's analysis was then verified by Krysta and Michelle. Thus, although we inscribe this project as what we learned, there was a delineation of roles: Krysta and Michelle enacted the action research project, while Paul mentored and interpreted Krysta and Michelle's learning using a model of the relation between theory and practice. Krysta and Michelle reported that Paul's interpretation

helped them organize and make sense of what they had learned, while Paul learned about inquiry vicariously through Krysta and Michelle.

Learning about Relations between Theory and Practice

This paper provides a representation of what we learned from this action research project, interpreted through the lens of a model of the relation between theory and practice of teaching. This model states that there are three relations between theory and practice: (1) applying theory to practice, (2) interpreting theory and practice, and (3) building/refining personal, practical and professional theories of teaching. In what follows, we describe these three theory-practice linkages, as sites of learning about inquiry using action research. We address each linkage in turn, beginning by illustrating what Krysta and Michelle learned, and then providing Paul's interpretation.

Applying the Theory of Inquiry to Practice

Michelle and Krysta summarized the theory of inquiry learning as follows. Inquiry is an open-ended teaching approach. The learning process has varying beginnings, middles, and ends and is the fusion of real world applications, curricular outcomes, and learning processes. Inquiry allows students to apply prior knowledge and creativity to satisfy their natural curiosities and make connections to curricular content. But learning also requires at least some structure in order to proceed. If the learning environment is too unstructured, learning potential may become eroded by chaos. But an inquiry need not be structured by specific outcomes or specific trajectories of learning.

Michelle and Krysta chose the "4D-Cycle Planning Model" (cf. Allmond, Wells, & Makar, 2010) as a structure for inquiry-based learning, which they summarize as follows. This model involves four dynamic phases of student learning, namely, discover, devise, develop, and defend. During the *discover* phase, the teacher introduces and motivates an essential question/problem, and students use their prior knowledge to develop their initial understandings. During the *devise* phase students are introduced to the main requirements of the inquiry and are required to come up with a plan for addressing those requirements. During the *develop* phase students implement their plan, developing and using mathematics to generate preliminary findings related to the inquiry. During the *defend* phase students present and justify their inquiry findings; and reflect on how they came to their solutions, what they could have done differently, and what new inquiry questions have arisen as a by-product of the initial inquiry. A fifth optional phase, *diverge*, can be enacted if the teacher or students wish to consider possible extensions of the inquiry.

Michelle and Krysta planned several inquiries using the 4D-Cycle Model, and implemented these inquiries. Examples of these inquiries were designing a room, creating a co-ordinate mapping system, researching a social justice issue, and surveying the perceptions of children from around the world. In the design a room inquiry, for example, the discover phase introduced the idea of creating a bedroom of your choice, the devise phase considered design parameters such as cost of painting the walls, the develop phase required students to use perimeter and area to ensure their room design satisfied specific parameters, and the students presented to the class either a 2D or 3D representation of their room in the defend phase.

Paul interpreted Krysta and Michelle's use of the 4D-Cycle Model as an example of applying theory to practice. They used the theory as-is to generate lesson plans. Such application is the norm in teacher education programs, where teachers are expected to use what they learn in university course work during their practicum experiences. In this case, the 4D-Cycle Model was found by Michelle and Krysta to solve a problem, namely, how to plan for inquiry. This is a legitimate application of theory to practice because it answered a question for Krysta and Michelle that was personally relevant to their development as teachers.

Interpreting the Theory and Practice of Inquiry

According to Krysta and Michelle, the flexibility of an inquiry-based learning environment allows students to learn in a manner best suited to their personal learning styles. As Small (2010) explains, despite recent changes in the manner in which math is taught in the early and middle years classrooms, there are still two conceptions that remain: first, that all students should be working on the same problem at the same time, and, second, that all students should come to one common answer. Inquiry moves beyond these conceptions and allows students to work in a manner that is best suited to them. This was the position of Krysta and Michelle at the end of the course.

During the practicum, Krysta and Michelle often reflected on the challenge of finding a balance between too much and too little structure for an inquiry. When the inquiry was too open-ended, students could be either overwhelmed to the point of inertia or tackle too much, so their learning was unfocused. When the inquiry was too structured, classroom dynamics tended toward reproducing traditional behaviors, thus undermining deeper degrees of inquiry. Further, Krysta and Michelle found that different groups needed different degrees of structure. For example, one group might need to be encouraged to focus their thinking while another needed extra prompting to continue with the inquiry.

By the end of the practicum, Krysta and Michelle had developed two pedagogical responses to the challenge of finding an appropriate balance between structure and openness. First, they encouraged risk-taking and continually reminded students that making mistakes was part of learning. Second, they resisted telling students how to proceed; rather, they scaffolded student thinking to maintain the momentum of an inquiry. For example, when a student asked, "is this right," they would respond with a "why" question (e.g., "Why did you do this calculation?"). This example illustrated both resisting telling an answer and encouraging risk-taking.

Paul interpreted Krysta and Michelle's learning as an example of interpreting the theory and practice of inquiry. Although others have theorized on the need for flexibility by teachers when using inquiry, Krysta and Michelle had not looked closely at this issue before the practicum. They noted in the course assignment, for example, that the Design a Room activity allowed students to use strategies that work best for them and to illustrate the model of their room in any manner they wished, without attention to the diversity of students or pedagogical requirements. Krysta and Michelle continued to interpret this piece of theory and their practice throughout the practicum. What does flexibility in inquiry mean? What does it mean to allow students to work in a manner that best suits them? What does it mean to resist highly structured conceptions of mathematics and mathematics

teaching? Krysta and Michelle answered these questions by responding to the challenge of finding an appropriate balance between structure and openness. Their interpretation of the theory of inquiry and their experience of inquiry is to find balance by resisting telling an answer and by encouraging risk taking. This is not applying theory to practice because these ideas were not theorized before the practicum. Rather, Krysta and Michelle, by responding to the in-situ needs of their practice, are interpreting the theory and practice of flexibility in inquiry.

Building/Refining Personal, Practical, and Professional Theories of Teaching Using Inquiry

According to Krysta and Michelle (taken from their assignment final report), inquiry-based learning moves away from the traditional conception of the math classroom where mathematics is conceived as a set of rules and processes to be learned and followed (Myers, 2007). A common conception of mathematics is that it consists of rules and theorems that are universally true. Mathematics curriculum documents tend to reinforce this conception because they can be interpreted as a list of skills that must be reproduced by students. Mathematics pedagogy can then devolve into largely transmission of knowledge and practice of skills. Inquiry as a method of teaching mathematics rejects transmission approaches, but then faces a more difficult task of ensuring that mandated curricular objectives are met. The difficulty is magnified by a rule-following conception of mathematics. A criticism of inquiry is that basic mathematical concepts and skills that must be learned are lost in the inquiry (Guan, 2010).

Krysta and Michelle faced this difficulty of inquiry throughout their action research project. They initially rejected this difficulty because inquiry triggered the natural curiosity of learners and because basic mathematical concepts and skills were tools needed to complete an inquiry. During their practicum, they still needed to motivate children to begin, continue or work more deeply on an inquiry. Krysta and Michelle found that some children who disliked math were immediately motivated by the openness of the inquiry, whereas others who tended to be successful in a rule-following mathematics classroom resisted the challenge of an inquiry. Krysta, in particular, felt pressure to resort to a rule-following approach when an inquiry would (temporarily) stall. Krysta and Michelle were also surprised by the successes of some students who traditionally struggled with mathematics. Further, they were encouraged by other teachers who saw evidence of the successes of their inquiry lessons.

In their final reflections, Krysta and Michelle affirmed their developing belief in inquiry as a method of teaching mathematics. They also recognized their own biases about what students should learn from an inquiry: if inquiry triggered learning according to the needs of each student, then teachers are not the final or only warranters of truth in the math class. Krysta justified inquiry by noting that the approach would prepare all students for all types of situations, including the traditional classrooms they would likely experience in secondary school. Michelle believed that teachers should move beyond thinking that they must “pass on” knowledge to students. Further, Krysta and Michelle adopted an inquiry approach even though they had to learn how to resist their desire for control of the learning process.

Paul interpreted Krysta and Michelle's learning as an example of building and refining a personal, practical and professional theory of teaching using inquiry. Their theory building involved the value of inquiry despite its challenges and criticisms, recognizing a need to shift their thinking from inquiry as application to inquiry as community-based knowledge construction, and developing a disposition of resisting trying to tightly control what is learned. This theory building is personal because it emerged with and from their inquiries into teaching using inquiry, and because they are committed to the potential effectiveness of inquiry despite the criticisms and barriers. It is practical because they faced the criticism concerning ensuring essential mathematics is learned and the barrier of rule-following conceptions of mathematics. In responding to this criticism and barrier, they developed teaching strategies, which can be applied each time they use an inquiry approach. Their learning is professional because it was informed by dialogue with educational professionals and mathematics education literature. Finally, they have built a theory because they will apply what they have learned in their future teaching and because others can learn from what they have learned.

Final Reflections

This action research project generated insight for us concerning the relationship between theory and practice and concerning the use of inquiry to teach mathematics. For Krysta and Michelle, their understandings of inquiry as a method of teaching math are enriched by applying the theory of inquiry to practice, by interpreting the theory and practice of inquiry, and developing their own personal, practical and professional theories of teaching using inquiry. In particular, they applied the 4D-Cycle Model to the task of planning for inquiry, they interpreted the flexible balance between structure and openness needed in inquiry, and they built a theory of teaching with inquiry based on non-linear and community-based dispositions of teachers toward learning. For Paul, Krysta's and Michelle's learning is further evidence that the relationship between theory and practice is indeed dynamic and non-linear, and that action research does trigger opportunities for teacher candidates to develop richer understandings of teaching by implicitly engaging with theoretical ideas situated in practice. In particular, we have illustrated that there are at least three modes by which teacher candidates can experience a linkage between theory and practice.

We appear to be categorizing linkages between theory and practice into three types. Interpreting theory and practice is not applying theory to practice because the interpretations were not a-priori theory from the perspective of the teacher candidates. Interpreting theory and practice also is not theory building/refining because it is an act of understanding rather than a creation of a new theory from the perspective of the learner. And theory building/refining is not applying theory to practice because the former emerges from practice while the latter occurs in practice.

It may be though that these interactions represent a developmental pathway, rather than modes of experiencing linkages between theory and practice. Applying theory to practice seems to be a universal consequence of polarizing theory and practice. With experience, teacher candidates can use what they learned to interpret their experience, rather than only view theory as ideas that might be applied, or not, to practice. Further, building/refining theory could require opportunities to both apply and interpret. That is,

interpretation and application are necessary conditions for the theory building/refining illustrated in this paper.

In our action research project, the use of the 4D-Cycle Model of inquiry by Krysta and Michelle (applying theory to practice) appears to be a necessary condition for further experiences of theory and practice. Subsequent to Krysta and Michelle's initial attempts to understand and use inquiry, they began to interpret connections between what they had read and what they were experiencing (interpreting theory and practice). Finally, these learning experiences served as a foundation for Krysta and Michelle to build a theory of teaching with inquiry that embraces a dynamic and collaborative notion of learning.

There is some evidence in the literature to support a developmental pathway interpretation. A case study by Cheng, Tang, and Cheng (2012) suggested a similar typology of "practicalising theoretical knowledge" may be developmental. Their three typologies are "procedural" (similar to applying theory to practice), "reflective-adaptive" (adapting theory based on reflection on practice), and "reflective-theorizing" (similar to theory building). They argued this typology could be developmental from procedural to reflective adaptive to reflective-theorizing because it represented a shift in the teacher candidate's attention from their teaching acts to also include the learning of children. Similarly, our notion of theory building is a richer and more nuanced attention to the complexity of teaching, formed by more sophisticated interactions between theoretical knowledge and practical experience.

Regardless of the relationship between the three types of linkages between theory and practice, it is clear that action research can trigger opportunities for teacher candidates to experience rich and non-linear interactions between the theory and practice of teaching. Paul designed conditions (the assignment) within which Krysta and Michelle's learning was complex. Like inquiry learning, action research cannot control what is learned; rather, it provides an opportunity for deeper learning because it is open-ended. Paul and other researchers could have told Krysta and Michelle, in theory, much of what they learned about inquiry. But this approach suffers from the problem of any teacher education environment that assumes applying theory to practice is sufficient. By moving beyond applying theory to practice, as occasioned by action research, Krysta and Michelle developed more nuanced understandings of inquiry learning in math, and Paul reaffirmed his belief in situated, collaborative and socially constructed theories of knowing and learning.

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